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# Weyl multiplets of conformal supergravity in five dimensions

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Abstract: We construct the Weyl multiplets of $N = 2$ conformal supergravity in five dimensions. We show that there exist two different versions of the Weyl multiplet, which contain the same gauge fields but differ in the matter field content: the Standard Weyl multiplet and the Dilaton Weyl multiplet. At the linearized level we obtain the transformation rules for the Dilaton Weyl multiplet by coupling it to the multiplet of currents corresponding to an on-shell vector multiplet. We construct the full non-linear transformation rules for both multiplets by gauging the $D = 5$ superconformal algebra $F^2(4)$. We show that the Dilaton Weyl multiplet can also be obtained by solving the equations of motion for an improved vector multiplet coupled to the Standard Weyl multiplet.

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## 1. Introduction

Conformal supergravities have been constructed in various dimensions (for a review, see [1]) but not yet in five dimensions. The five-dimensional case is of interest for various reasons not least of all from a purely mathematical viewpoint since it is based on the exceptional superalgebra $F^2(4)$.

By using conformal tensor calculus, conformal supergravities form an elegant way to construct general couplings of Poincaré-supergravities to matter [2]. In the five-dimensional case these matter coupled supergravities have recently attracted renewed attention due to the important role they play in the Randall–Sundrum (RS) scenario [3, 4] and the $AdS_6/CFT_5$ [5, 6] and $AdS_5/CFT_4$ [7] correspondences.

The form of the scalar potential in five-dimensional matter coupled supergravities plays a crucial role in the possible supersymmetrisation of the RS-scenario. It turns out that such a supersymmetrisation is non-trivial. With only vector multiplets and no singular source insertions, a no-go theorem was established for smooth domain-wall solutions [8, 9]. In view of this, general $D = 5$ supergravity/matter couplings have been re-investigated [10], thereby generalizing the earlier results of [11, 12]. A modification of the theory allows solutions by inserting branes as singular insertions [13]. The inclusion of hypermultiplets was first considered in [14], where even generalizations of [10] were considered. However, this description has not been proven to be consistent. Hypermultiplets were also considered in [15, 16]. The mixing of vector and hypermultiplets [17] seems to circumvent all obstructions, though no example of a good smooth solution has been found. However, it has been shown also in [17] that $N = 2$, $D = 5$ matter couplings to supergravity can give rise to more general possibilities for renormalization group flows between conformal theories in ultraviolet and infrared than those known for $N = 8$.

With all these developments, it is clear that it is important that there is an independent derivation of the most general matter couplings derived in [10]. Moreover, it has turned out in the past that superconformal constructions lead to insights in the structure of matter couplings. A recent example is the insight in relations between hyper-Kähler cones and quaternionic manifolds, based on the study of superconformal invariant matter couplings with hypermultiplets [18]. For all these reasons a superconformal construction of general matter couplings in $N = 2$, $D = 5$ is useful.

In this paper we take the first step in this investigation by constructing the $N = 2$, $D = 5$ conformal supergravity theory. In our construction we use the methods developed first for $N = 1$, $D = 4$ [19, 20]. They are based on gauging the conformal superalgebra [21] which in our case is $F^2(4)$.

The superconformal multiplet that contains all the (independent) gauge fields of the superconformal algebra is called the Weyl multiplet. In general one needs to include matter fields to have an equal number of bosons and fermions. We will see that in five dimensions there are two possible sets of matter fields one can add, yielding two versions of the Weyl multiplet: the Standard Weyl multiplet and the Dilaton Weyl multiplet. This result is similar to what was found for $(1, 0)$ $D = 6$ conformal supergravity theory [22]. Also in that case, two versions were found: a multiplet containing a dilaton and one without a dilaton.

In [5], the field content and transformation rules for the Standard Weyl multiplet were constructed from the $F(4)$-gauged six-dimensional supergravity using the

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$AdS_6/CFT_5$ correspondence. The results, although not given in a manifestly superconformal notation, seem similar to ours. However, the full non-linear commutation relations (see (4.3)) that we obtain were not given.

Another attempt was undertaken in [23] by reducing the known six-dimensional result [22] to five dimensions. The authors of [23] already gauge-fixed some symmetries of the superconformal algebra during the reduction process in order to simplify the matter multiplet coupling. In this way, they found a multiplet that is larger than the Weyl multiplet that we will construct in this work, because they do not aim to obtain superconformal symmetry in 5 dimensions. Our strategy is to start from the basic building blocks of superconformal symmetry in 5 dimensions.

We first construct the conformal supercurrent multiplet that contains the energy–momentum tensor of the $D = 5$ vector multiplet. This is non-trivial because the $D = 5$ vector multiplet is *not* conformal. At first sight this seems to prohibit the construction of a *conformal* current multiplet, but we will show how the introduction of a dilaton in the Weyl multiplet circumvents this obstacle. This is the origin of the first of our two versions of 32+32 Weyl multiplets. The other one is a straightforward extension of the one known in 4 dimensions.

We have organized the paper such that a reader who is interested in the main results for the multiplets, i.e. their content, transformation laws and the algebra that they satisfy can find everything in section 4. The rules found in this section will be needed when one investigates matter couplings. However, this does not contain all our results. The relation between the two versions is based on the use of the (improved) vector multiplet, and this construction is also part of our main result.

This paper is organized as follows. In section 2, as the first step in our procedure, we construct the supercurrent multiplet that contains the energy-momentum tensor of the $N = 2$, $D = 5$ vector multiplet. It turns out that this supercurrent multiplet has $32 + 32$ components.

The coupling of the supercurrent multiplet to the fields of conformal supergravity leads to the linearized superconformal transformation rules for the 32+32 component Dilaton Weyl multiplet. We show that there exists another version of the linearized Weyl multiplet (the Standard Weyl multiplet) that contains the same gauge fields as the Dilaton Weyl multiplet, but differs in the matter field content. An important difference between the Standard and Dilaton Weyl multiplet is that the scalar field of the Standard Weyl multiplet has a non-zero mass dimension that cannot serve, like the dilaton scalar field of the Dilaton Weyl multiplet, as a compensator for scale transformations.

In section 3 we derive the full non-linear transformation rules for both Weyl multiplets by gauging the $D = 5$ superconformal algebra $F^2(4)$ following the notations on real forms as in [24]. For the convenience of the reader we give the final results of the two Weyl multiplets, in a self-contained manner, in section 4.

In section 5 we show that the Dilaton Weyl multiplet can be obtained by coupling the Standard Weyl multiplet to an improved vector multiplet. This establishes the precise connection between the two multiplets. We present our conclusions in section 6.

We explain our notation and conventions in appendix A. The complete commutation relations defining the $D = 5$ superconformal algebra $F^2(4)$ are given in appendix B. Finally, in appendix C we compare the $32 + 32$ supercurrent multiplet we construct in this paper with the $40 + 40$ supercurrent multiplet constructed by Howe and Lindström [25] some time ago. We show that their multiplet is reducible.

## 2. Linearized Weyl multiplets

In this section we obtain two linearized Weyl multiplets. After discussing the method of the supercurrent (section 2.1) we will construct the currents of a rigid on-shell vector multiplet (section 2.2), and define a Weyl multiplet as the fields that couple to the currents (section 2.3). The comparison with known Weyl multiplets in 4 and 6 dimensions, tells us that there is also another Weyl multiplet, and we point out that it can be obtained from the first one by redefining some fields (section 2.4).

### 2.1 The current multiplet method

The multiplet of currents in a superconformal context has been discussed before in the literature, e.g. the current multiplet corresponding to the $N = 1$, $D = 4$ [20], the $N = 2$, $D = 4$ [26, 27] and the $N = 4$, $D = 4$ vector multiplets [28] and to the (self-dual) $(2, 0)$ $D = 6$ tensor multiplet [29].

After adding local improvement terms one obtains a supercurrent multiplet containing an energy-momentum tensor $\theta_{\mu\nu} = \theta_{\nu\mu}$ and a supercurrent $J^i_\mu$ which are both conserved and (gamma-)traceless

$$\partial^\mu \theta_{\mu\nu} = \theta_\mu{}^\mu = \partial^\mu J^i_\mu = \gamma^\mu J^i_\mu = 0\,. \tag{2.1}$$

These improved current multiplets were used in the past to construct the linearized transformation rules for the Weyl multiplet[1] since a traceless energy-momentum tensor is equivalent to scale-invariance of the kinetic terms in the action.

However, the standard kinetic term of the $D = 5$ vector field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{2.2}$$

is not scale invariant, i.e. the energy-momentum tensor is not traceless:

$$\theta_{\mu\nu} = -F_{\mu\lambda} F_\nu{}^\lambda + \frac{1}{4}\eta_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}\,, \qquad\qquad \theta_\mu{}^\mu = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \neq 0\,. \tag{2.3}$$

Moreover, there do not exist gauge-invariant local improvement terms.

[1]The Weyl multiplets of $(1,0)$ $D = 6$ [22] were derived without the use of a current multiplet, although this is certainly possible in view of the reduction rules given in [29].



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There is a remedy for this problem. Whenever there is a compensating scalar field present, i.e. a scalar with mass dimension zero but non-zero Weyl weight, then the kinetic term (2.2) can be made scale invariant by introducing a scalar coupling of the form

$$\mathcal{L} = -\frac{1}{4} e^{\phi} F_{\mu\nu} F^{\mu\nu} \, . \tag{2.4}$$

This compensating scalar is called the dilaton. In general, there are three possible origins for a dilaton coupling to a non-conformal matter multiplet: the dilaton is part of

1. the matter multiplet itself (the multiplet is then called an 'improved' multiplet);

2. the conformal supergravity multiplet;

3. another matter multiplet.

The $N = 2$, $D = 5$ vector multiplet contains precisely such a scalar. We could therefore use it to compensate the broken scale invariance of the kinetic terms. This leads to the so-called improved vector multiplet. This is the first possibility, that will be further discussed in section 5.

The second possibility will be considered here (the third possibility is included for completeness). This possibility thus occurs when the Weyl multiplet itself contains a dilaton. We will see that there indeed exists a version of the Weyl multiplet containing a dilaton. This version is called the Dilaton Weyl multiplet. It turns out that there exists another version of the Weyl multiplet without a dilaton. This other version will be called the Standard Weyl multiplet.

For matter multiplets having a traceless energy-momentum tensor, no compensating scalar is needed. To see the difference between the various cases it is instructive to consider $(1,0)$ $D = 6$ conformal supergravity theory [22] which was constructed without the supercurrent method. In that case, two versions were found: a multiplet containing a dilaton and one without a dilaton. We expect that both versions can be constructed using the supercurrent method: the one without a dilaton starting from the conformal $(1,0)$ tensor multiplet (being a truncation of the $(2,0)$ case), and the version containing the dilaton by starting from the non-conformal $D = 6$ vector multiplet (which upon reduction should produce our results in $D = 5$).

Thus, the current multiplet needs to be improved only when coupled to the Standard Weyl multiplet. In the case of the Dilaton Weyl multiplet it is not necessary to do so, since in that case the dilaton of the Weyl multiplet can be used to compensate for the lack of scale invariance. In particular, the dilaton will couple directly to the trace of the energy-momentum tensor.

When coupling to the Standard Weyl multiplet one needs to add *non-local* improvement terms to the current multiplet which was done for the current multiplet

| Field | Equation of motion | SU(2) | $w$ | # d.o.f. |
|---|---|---|---|---|
| $A_\mu$ | $\partial_\mu F^{\mu\nu} = 0$ | 1 | 0 | 3 |
| $\sigma$ | $\Box\sigma = 0$ | 1 | 1 | 1 |
| $\psi^i$ | $\not{\partial}\psi^i = 0$ | 2 | 3/2 | 4 |

**Table 1:** The $4+4$ on-shell abelian vector multiplet.

coming from the $D = 10$ vector multiplet [30]. In that case the non-local improvement terms that were added, required the use of auxiliary fields satisfying differential constraints in order to make the transformation rules local.[2]

We did not analyse the addition of non-local counter terms. It would be interesting to see if in this way a consistent coupling to the Standard Weyl multiplet can be obtained. Instead, we will derive the linearized transformation rules for the Standard Weyl multiplet via a field redefinition from those of the Dilaton Weyl multiplet.

### 2.2 Current multiplet of the $N = 2$, $D = 5$ vector multiplet

Our starting point is the on-shell $D = 5$ vector multiplet. Its field content is given by a massless vector $A_\mu$, a symplectic Majorana spinor $\psi^i$ in the fundamental of SU(2) and a real scalar $\sigma$. See table 1 for additional information. Our conventions are given in appendix A.

The action for the $D = 5$ Maxwell multiplet is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\bar{\psi}\not{\partial}\psi - \frac{1}{2}(\partial\sigma)^2 . \tag{2.5}$$

This action is invariant under the following supersymmetries

$$\begin{aligned}
\delta_Q A_\mu &= \frac{1}{2}\bar{\epsilon}\gamma_\mu\psi \,, \\
\delta_Q \psi^i &= -\frac{1}{4}\gamma\cdot F\epsilon^i - \frac{1}{2}\,\mathrm{i}\,\not{\partial}\sigma\epsilon^i \,, \\
\delta_Q \sigma &= \frac{1}{2}\,\mathrm{i}\,\bar{\epsilon}\psi \,,
\end{aligned} \tag{2.6}$$

as well as under the standard gauge transformation

$$\delta_\Lambda A_\mu = \partial_\mu\Lambda \,. \tag{2.7}$$

The various symmetries of the lagrangian (2.5) lead to a number of Noether currents: the energy-momentum tensor $\theta_{\mu\nu}$, the supercurrent $J^i_\mu$ and the SU(2)-current $v^{ij}_\mu$. The supersymmetry variations of these currents lead to a closed multiplet of

[2]Note also that in $D = 10$ the trace-part and the traceless part of the energy-momentum tensor are not contained in the same multiplet which necessitates the addition of the non-local improvement terms to project out the trace-part.



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| Current | Noether | SU(2) | $w$ | # d.of. |
|---|---|---|---|---|
| $\theta_{(\mu\nu)}$ | $\partial^\mu\theta_{\mu\nu}=0$ | 1 | 2 | 9 |
| ${\theta_\mu}^\mu$ | | 1 | 4 | 1 |
| $v_\mu^{(ij)}$ | $\partial^\mu v_\mu^{ij}=0$ | 3 | 2 | 12 |
| $a_\mu$ | $\partial^\mu a_\mu=0$ | 1 | 3 | 4 |
| $b_{[\mu\nu]}$ | $\partial^\mu b_{\mu\nu}=0$ | 1 | 2 | 6 |
| $J_\mu^i$ | $\partial^\mu J_\mu^i=0$ | 2 | 5/2 | 24 |
| $\zeta^i\equiv \mathrm{i}\,\gamma\cdot J^i$ | | 2 | 7/2 | 8 |

**Table 2:** The $32+32$ current multiplet. The trace ${\theta_\mu}^\mu$ and the gamma-trace of $J_\mu^i$ form separate currents, the latter is denoted by $\zeta^i$.

$32+32$ degrees of freedom (see table 2). As discussed in the introduction, an unconventional feature, compared to the currents corresponding to a $D=4$ vector multiplet or a $D=6$ tensor multiplet, is that the current multiplet cannot be improved by local gauge-invariant terms, i.e. ${\theta_\mu}^\mu\neq 0$ and $\gamma^\mu J_\mu^i\neq 0$. It is convenient to include these trace parts as separate currents since, as it turns out, they couple to independent fields of the Weyl multiplet.

We find the following expressions for the Noether currents and their supersymmetric partners in terms of bilinears of the vector multiplet fields:

$$\begin{aligned}
\theta_{\mu\nu} &= -\partial_\mu\sigma\partial_\nu\sigma+\frac{1}{2}\eta_{\mu\nu}\,(\partial\sigma)^2-F_{\mu\lambda}{F_\nu}^\lambda+\frac{1}{4}\eta_{\mu\nu}F^2-\frac{1}{2}\bar\psi\gamma_{(\mu}\partial_{\nu)}\psi\,,\\
J_\mu^i &= -\frac{1}{4}\,\mathrm{i}\,\gamma\cdot F\gamma_\mu\psi^i-\frac{1}{2}(\partial\!\!\!/\sigma)\gamma_\mu\psi^i\,,\\
v_\mu^{ij} &= \frac{1}{2}\bar\psi^i\gamma_\mu\psi^j\,,\\
a_\mu &= \frac{1}{8}\varepsilon_{\mu\nu\lambda\rho\sigma}F^{\nu\lambda}F^{\rho\sigma}+(\partial^\nu\sigma)F_{\nu\mu}\,,\\
b_{\mu\nu} &= \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho\sigma}(\partial^\lambda\sigma)F^{\rho\sigma}+\frac{1}{2}\bar\psi\gamma_{[\mu}\partial_{\nu]}\psi\,,\\
\zeta^i &= \mathrm{i}\,\gamma\cdot J^i=\frac{1}{4}\gamma\cdot F\psi^i+\frac{3}{2}\,\mathrm{i}\,\partial\!\!\!/\sigma\psi^i\,,\\
{\theta_\mu}^\mu &= \frac{3}{2}\,(\partial\sigma)^2+\frac{1}{4}F^2\,.
\end{aligned}\tag{2.8}$$

From these expressions, using the Bianchi identities and equations of motion of the vector multiplet fields, one can calculate the supersymmetry transformations of the currents. A straightforward calculation yields:

$$\begin{aligned}
\delta_Q\theta_{\mu\nu} &= \frac{1}{2}\,\mathrm{i}\,\bar\epsilon\gamma_{\lambda(\mu}\partial^\lambda J_{\nu)}\,,\\
\delta_Q J_\mu^i &= -\frac{1}{2}\,\mathrm{i}\,\gamma^\nu\theta_{\mu\nu}\epsilon^i-\mathrm{i}\,\gamma_{[\lambda}\partial^\lambda v_{\mu]}^{ij}\epsilon_j-\frac{1}{2}a_\mu\epsilon^i+\frac{1}{2}\,\mathrm{i}\,\gamma^\nu b_{\mu\nu}\epsilon^i\,,
\end{aligned}$$

$$
\begin{aligned}
\delta_Q v_\mu^{ij} &= \mathrm{i}\,\bar\epsilon^{(i} J_\mu^{j)}\,,\\
\delta_Q a_\mu &= -\bar\epsilon\partial^\lambda\gamma_{[\lambda}J_{\mu]} + \frac{1}{4}\bar\epsilon\gamma_{\rho\mu}\gamma^\sigma\partial^\rho J_\sigma + \frac{1}{4}\,\mathrm{i}\,\bar\epsilon\gamma_{\rho\mu}\partial^\rho\zeta\,,\\
\delta_Q b_{\mu\nu} &= \frac{3}{4}\,\mathrm{i}\,\bar\epsilon\gamma_{[\lambda\mu}\partial^\lambda J_{\nu]} - \frac{1}{8}\,\mathrm{i}\,\bar\epsilon\gamma_{\rho\mu\nu}\gamma^\lambda\partial^\rho J_\lambda + \frac{1}{8}\bar\epsilon\gamma_{\rho\mu\nu}\partial^\rho\zeta\,,\\
\delta_Q\zeta^i &= \frac{1}{2}\rho\epsilon^i - \frac{1}{2}\partial\!\!\!/\psi^{ij}\epsilon_j - \frac{1}{2}\,\mathrm{i}\,a\!\!\!/\epsilon^i - \frac{1}{2}\gamma\cdot b\epsilon^i\,,\\
\delta_Q\theta_\mu{}^\mu &= \frac{1}{2}\bar\epsilon\partial\!\!\!/\zeta\,.
\end{aligned}
\tag{2.9}
$$

Note that we have added to the transformation rules for $a_\mu$ and $b_{\mu\nu}$ terms that are identically zero: the first term at the r.h.s. contains the divergence of the supercurrent and the last two terms are proportional to the combination $(i\gamma\cdot J-\zeta)$ which is zero. Similarly, the second term in the variation of the supercurrent contains a term that is proportional to the divergence of the SU(2) current.

The reason why we added these terms is that in this way we obtain below the linearized Weyl multiplet in a conventional form. Alternatively, we could not have added these terms and later have brought the Weyl multiplet into the same conventional form by redefining the $Q$-transformations via a field-dependent $S$- and SU(2)-transformation.

### 2.3 Linearized Dilaton Weyl multiplet

The linearized $Q$-supersymmetry transformations of the Weyl multiplet are determined by coupling every current to a field, and demanding invariance of the corresponding action. The field-current action is given by:

$$
S = \int d^5x\Big(\frac{1}{2}h_{\mu\nu}\theta^{\mu\nu} + \mathrm{i}\,\bar\psi_\mu J^\mu + V_\mu^{ij}v^\mu_{ij} + A_\mu a^\mu + B_{\mu\nu}b^{\mu\nu} + \mathrm{i}\,\bar\psi\zeta + \varphi\ \theta_\mu{}^\mu\Big)\,. \tag{2.10}
$$

In table 3 we give some properties of the Weyl multiplets. In particular of the one just derived, which we call the Dilaton Weyl multiplet[3]. A similar Weyl multiplet containing a dilaton exists in $D=6$ [22].

Using the supersymmetry rules for the current multiplet, we find that the following transformations leave the action (2.10) invariant:

$$
\begin{aligned}
\delta_Q h_{\mu\nu} &= \bar\epsilon\gamma_{(\mu}\psi_{\nu)}\,,\\
\delta_Q\psi_\mu^i &= -\frac{1}{4}\gamma^{\lambda\nu}\partial_\lambda h_{\nu\mu}\epsilon^i - V_\mu^{ij}\epsilon_j + \frac{1}{8}\,\mathrm{i}\Big(\gamma\cdot F + \frac{1}{3}\,\mathrm{i}\,\gamma\cdot H\Big)\gamma_\mu\epsilon^i\,,\\
\delta_Q V_\mu^{ij} &= -\frac{1}{2}\bar\epsilon^{(i}\gamma^\lambda\psi^{j)}_{\lambda\mu} + \frac{1}{2}\,\mathrm{i}\,\bar\epsilon^{(i}\gamma_\mu\partial\!\!\!/\psi^{j)}\,,
\end{aligned}
$$

[3]Note that the Dilaton Weyl multiplet contains a vector $A_\mu$, a spinor $\psi^i$ and a scalar $\sigma$ which, on purpose, we have given the same names as the fields of the vector multiplet. The reason for doing so will become clear in section 5 where we explain the connection between the two Weyl multiplets. From now on, until section 5, we will be only dealing with the Weyl multiplets and not with the vector multiplet. Therefore, our notation should not lead to confusion.

| Field | # | Gauge | SU(2) | $w$ | Field | # | Gauge | SU(2) | $w$ |
|---|---|---|---|---|---|---|---|---|---|
| | | Elementary gauge fields | | | | | Dependent gauge fields | | |
| $e_\mu{}^a$ | 9 | $P^a$ | 1 | $-1$ | $\omega_\mu^{[ab]}$ | – | $M^{[ab]}$ | 1 | 0 |
| $b_\mu$ | 0 | $D$ | 1 | 0 | $f_\mu{}^a$ | – | $K^a$ | 1 | 1 |
| $V_\mu^{(ij)}$ | 12 | SU(2) | 3 | 0 | | | | | |
| $\psi_\mu^i$ | 24 | $Q_\alpha^i$ | 2 | $-1/2$ | $\phi_\mu^i$ | – | $S_\alpha^i$ | 2 | $1/2$ |
| | | Dilaton Weyl multiplet | | | | | Standard Weyl multiplet | | |
| $A_\mu$ | 4 | $\delta A_\mu = \partial_\mu \Lambda$ | 1 | 0 | $T_{[ab]}$ | 10 | | 1 | 1 |
| $B_{[\mu\nu]}$ | 6 | $\delta B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}$ | 1 | 0 | | | | | |
| $\varphi$ | 1 | | 1 | 1 | $D$ | 1 | | 1 | 2 |
| $\psi^i$ | 8 | | 2 | $3/2$ | $\chi^i$ | 8 | | 2 | $3/2$ |

**Table 3:** Fields of the Weyl multiplets, and their roles. The upper half contains the fields that are present in all versions. They are the gauge fields of the superconformal algebra (see section 3). The fields at the right-hand side of the upper half are dependent fields, and are not visible in the linearized theories. The symbol # indicates the off-shell degrees of freedom. The gauge degrees of freedom corresponding to the gauge invariances of the right half are subtracted from the fields at the left on the same row. In the lower half are the extra matter fields that appear in the two versions of the Weyl multiplet. In the left half are those of the Dilaton Weyl multiplet, at the right are those of the Standard Weyl multiplet. We also indicated the (generalized) gauge symmetries of the fields $A_\mu$ and $B_{\mu\nu}$. (The linearized fields, corresponding to $e_\mu{}^a$ and $\sigma \equiv e^\phi$ are denoted by $h_\mu{}^a$ and $\varphi$, respectively.)

$$\begin{aligned}
\delta_Q A_\mu &= -\frac{1}{2}\,\mathrm{i}\,\bar\epsilon\psi_\mu + \frac{1}{2}\bar\epsilon\gamma_\mu\psi\,,\\
\delta_Q B_{\mu\nu} &= \frac{1}{2}\bar\epsilon\gamma_{[\mu}\psi_{\nu]} + \frac{1}{2}\,\mathrm{i}\,\bar\epsilon\gamma_{\mu\nu}\psi\,,\\
\delta_Q \psi^i &= -\frac{1}{8}\gamma\cdot F\epsilon^i - \frac{1}{2}\,\mathrm{i}\,\partial\!\!\!/\varphi\epsilon^i + \frac{1}{24}\,\mathrm{i}\,\gamma\cdot H\epsilon^i\,,\\
\delta_Q \varphi &= \frac{1}{2}\,\mathrm{i}\,\bar\epsilon\psi\,,
\end{aligned} \tag{2.11}$$

where we have defined

$$F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}, \quad H_{\mu\nu\lambda} = 3\partial_{[\mu}B_{\nu\lambda]}, \quad \psi_{\mu\nu} = 2\partial_{[\mu}\psi_{\nu]}\,. \tag{2.12}$$

### 2.4 Linearized Standard Weyl multiplet

It turns out that there exists a second formulation of the Weyl multiplet in which the fields $A_\mu$ and $B_{\mu\nu}$ are replaced by an anti-symmetric tensor $T_{ab}$ and where also the spinor and the scalar are redefined. It is the multiplet we should have expected if we compare it with the Weyl multiplets of the $D=4$ and $D=6$ theories with 8 supercharges. This can be seen in table 4.

This second Weyl multiplet is called the Standard Weyl multiplet. More information about the component fields can be found in table 3. The Standard Weyl multiplet cannot be obtained from the same current multiplet procedure we applied to get the Dilaton Weyl multiplet, unless we would consider an 'improved' current multiplet. The reason is that the Standard Weyl multiplet contains no dilaton scalar with a zero mass dimension that can be used as a compensating scalar. Therefore it can not define a conformal coupling to a non-improved current multiplet.

| Field | $d=4$ | $d=5$ | $d=6$ |
|---|---|---|---|
| $e_\mu{}^a$ | 5 | 9 | 14 |
| $b_\mu$ | 0 | 0 | 0 |
| $\omega_\mu{}^{ab}$ | – | – | – |
| $f_\mu{}^a$ | – | – | – |
| $V_{\mu i}{}^j$ | 9 | 12 | 15 |
| $A_\mu$ | 3 | – | – |
| $\psi_\mu{}^i$ | 16 | 24 | 32 |
| $\phi_\mu{}^i$ | – | – | – |
| $T_{ab}, T^-_{abc}$ | 6 | 10 | 10 |
| $D$ | 1 | 1 | 1 |
| $\chi^i$ | 8 | 8 | 8 |
| TOTAL | $24+24$ | $32+32$ | $40+40$ |

**Table 4:** Number of components in the fields of the Standard Weyl multiplet. The dependent fields have no number. The field $T$ is a two rank tensor in 4 dimensions and a self-dual three rank tensor in 6 dimensions. In 5 dimensions we can choose between a two-rank or a three-rank tensor as these are dual to each other.

In 5 dimensions the full superconformal algebra cannot be realized on a matter multiplet without 'improvement' by a dilaton. In section 5 we will explain how the two Weyl multiplets can be related to each other via the coupling of the Standard Weyl multiplet to an improved vector multiplet.

The connection between the two versions of the Weyl multiplet at the linearized level is given by algebraic relations. First of all we denote some particular terms in the transformations of $\psi^i_\mu$ and $V^{ij}_\mu$ by $T_{ab}$ and $\chi^i$. Then we compute the variations of these expressions under supersymmetry, finding one more object called $D$. We find

$$\begin{aligned} T_{ab} &= \frac{1}{8}\Big(F_{ab} - \frac{1}{6}\varepsilon_{abcde}H^{edc}\Big)\,, \\ \chi^i &= \frac{1}{8}\,\mathrm{i}\,\partial\!\!\!/\psi^i + \frac{1}{64}\gamma^{ab}\psi_{ab}\,, \\ D &= \frac{1}{4}\Box\varphi - \frac{1}{32}\partial^\mu\partial^\nu h_{\mu\nu} + \frac{1}{32}\Box h^\mu_\mu\,. \end{aligned} \tag{2.13}$$

The resulting supersymmetry transformations are those of what we call the linearized Standard Weyl multiplet. They are given by

$$\begin{aligned} \delta_Q h_{\mu\nu} &= \bar\epsilon\gamma_{(\mu}\psi_{\nu)}\,, \\ \delta_Q\psi^i_\mu &= -\frac{1}{4}\gamma^{\lambda\nu}\partial_\lambda h_{\nu\mu}\epsilon^i - V^{ij}_\mu\epsilon_j + \mathrm{i}\,\gamma\cdot T\gamma_\mu\epsilon^i\,, \end{aligned}$$

| Generators | $P_a$ | $M_{ab}$ | $D$ | $K_a$ | $U_{ij}$ | $Q_{\alpha i}$ | $S_{\alpha i}$ |
|---|---|---|---|---|---|---|---|
| Fields | ${e_\mu}^a$ | $\omega_\mu^{ab}$ | $b_\mu$ | ${f_\mu}^a$ | $V_\mu^{ij}$ | $\psi_\mu^i$ | $\phi_\mu^i$ |
| Parameters | $\xi^a$ | $\lambda^{ab}$ | $\Lambda_D$ | $\Lambda_K^a$ | $\Lambda^{ij}$ | $\epsilon^i$ | $\eta^i$ |

**Table 5:** The gauge fields and parameters of the superconformal algebra $F^2(4)$.

$$\begin{aligned}
\delta_Q V_\mu^{ij} &= -\frac{1}{8}\bar{\epsilon}^{(i}\Big(\gamma^{ab}\gamma_\mu - \frac{1}{2}\gamma_\mu\gamma^{ab}\Big)\psi_{ab}^{j)} + 4\bar{\epsilon}^{(i}\gamma_\mu\chi^{j)}\,,\\
\delta_Q T_{ab} &= \frac{1}{2}\,\mathrm{i}\,\bar{\epsilon}\gamma_{ab}\chi - \frac{3}{32}\,\mathrm{i}\,\bar{\epsilon}\Big(\psi_{ab} - \frac{1}{12}\gamma_{ab}\gamma^{cd}\psi_{cd} + \frac{2}{3}\gamma_{[a}\gamma^c\psi_{b]c}\Big)\,,\\
\delta_Q\chi^i &= \frac{1}{4}D\epsilon^i - \frac{1}{64}\gamma^{\mu\nu}V_{\mu\nu}^{ij}\epsilon_j + \frac{3}{32}\,\mathrm{i}\,\gamma\cdot T\overleftarrow{\not{\partial}}\,\epsilon^i + \frac{1}{32}\,\mathrm{i}\,\not{\partial}\gamma\cdot T\epsilon^i\,,\\
\delta_Q D &= \bar{\epsilon}\not{\partial}\chi\,,
\end{aligned} \tag{2.14}$$

where we have defined

$$V_{\mu\nu}^{ij} = 2\partial_{[\mu}V_{\nu]}^{ij}\,. \tag{2.15}$$

This concludes our discussion of the linearized Weyl multiplets.

## 3. Gauging the superconformal algebra

We now proceed with the construction of the full Weyl multiplets, of which we have shown so far the linearized structure. We apply the methods developed first for $N = 1$ in 4 dimensions [20]. They are based on gauging the conformal superalgebra [21], which, in our case, is $F^2(4)$. The commutation relations defining the $F^2(4)$ algebra are given in appendix B. We first discuss the general method, and then apply this to construct the full (non-linear) Weyl multiplets for both versions that we found at the linearized level in section 2. For clarity, we have collected the final results in section 4.

### 3.1 The gauge fields and their curvatures

The $D = 5$ conformal supergravity theory is based on the superconformal algebra $F^2(4)$ whose generators are those in table 5, where $a, b, \ldots$ are Lorentz indices, $\alpha$ is a spinor index and $i = 1, 2$ is an SU(2) index. $M_{ab}$ and $P_a$ are the Poincaré generators, $K_a$ is the special conformal transformation, $D$ the dilatation, $Q_{i\alpha}$ and $S_{i\alpha}$ are the supersymmetry and the special supersymmetry generators, respectively, which are symplectic Majorana spinors, 8 real components in total. Finally, $U^{ij} = U^{ji}$ are the SU(2) generators. For more details on the $F^2(4)$ algebra and the rigid superconformal transformations, see [24]. The commutation relations of the generators are given in appendix B.

As a first step we assign to every generator of the superconformal algebra a gauge field. These gauge fields and the names of the corresponding gauge parameters are given in table 5.

The transformations are generated by operators according to

$$\delta = \xi^a P_a + \lambda^{ab} M_{ab} + \Lambda_D D + \Lambda_K^a K^a + \Lambda^{ij} U_{ij} + \mathrm{i}\,\bar\epsilon Q + \mathrm{i}\,\bar\eta S\,. \tag{3.1}$$

The i factors in the last two terms appear due to the reality properties, as explained in appendix A.

We can read off the transformation rules for the gauge fields from the algebra (B.1) using the general rules for gauge theories. We find

$$\begin{aligned}
\delta e_\mu{}^a &= \mathcal{D}_\mu \xi^a - \lambda^{ab} e_{\mu b} - \Lambda_D e_\mu{}^a + \frac{1}{2}\bar\epsilon\gamma^a\psi_\mu\,,\\
\delta \omega_\mu{}^{ab} &= \mathcal{D}_\mu \lambda^{ab} - 4\xi^{[a} f_\mu{}^{b]} - 4\Lambda_K^{[a} e_\mu{}^{b]} + \frac{1}{2}\,\mathrm{i}\,\bar\epsilon\gamma^{ab}\phi_\mu - \frac{1}{2}\,\mathrm{i}\,\bar\eta\gamma^{ab}\psi_\mu\,,\\
\delta b_\mu &= \partial_\mu \Lambda_D - 2\xi^a f_{\mu a} + 2\Lambda_K^a e_{\mu a} + \frac{1}{2}\,\mathrm{i}\,\bar\epsilon\phi_\mu + \frac{1}{2}\,\mathrm{i}\,\bar\eta\psi_\mu\,,\\
\delta f_\mu{}^a &= \mathcal{D}_\mu \Lambda_K^a - \lambda^{ab} f_{\mu b} + \Lambda_D f_\mu{}^a + \frac{1}{2}\bar\eta\gamma^a\phi_\mu\,,\\
\delta V_\mu^{ij} &= \partial_\mu \Lambda^{ij} - 2\Lambda^{(i}{}_\ell V_\mu^{j)\ell} - \frac{3}{2}\,\mathrm{i}\,\bar\epsilon^{(i}\phi_\mu^{j)} + \frac{3}{2}\,\mathrm{i}\,\bar\eta^{(i}\psi_\mu^{j)}\,,\\
\delta \psi_\mu^i &= \mathcal{D}_\mu \epsilon^i + \mathrm{i}\,\xi^a\gamma_a\phi_\mu^i - \frac{1}{4}\lambda^{ab}\gamma_{ab}\psi_\mu^i - \frac{1}{2}\Lambda_D\psi_\mu^i - \Lambda^i{}_j\psi_\mu^j - \mathrm{i}\,e_\mu^a\gamma_a\eta^i\,,\\
\delta \phi_\mu^i &= \mathcal{D}_\mu \eta^i - \frac{1}{4}\lambda^{ab}\gamma_{ab}\phi_\mu^i + \frac{1}{2}\Lambda_D\phi_\mu^i - \Lambda^i{}_j\phi_\mu^j - \mathrm{i}\,\Lambda_K^a\gamma_a\psi_\mu^i + \mathrm{i}\,f_\mu^a\gamma_a\epsilon^i\,,
\end{aligned} \tag{3.2}$$

where $\mathcal{D}_\mu$ is the covariant derivative with respect to dilatations, Lorentz rotations and SU(2) transformations:

$$\begin{aligned}
\mathcal{D}_\mu \xi^a &= \partial_\mu \xi^a + b_\mu \xi^a + \omega_\mu{}^{ab}\xi_b\,,\\
\mathcal{D}_\mu \lambda^{ab} &= \partial_\mu \lambda^{ab} + 2\omega_{\mu c}{}^{[a}\lambda^{b]c}\,,\\
\mathcal{D}_\mu \Lambda_K^a &= \partial_\mu \Lambda_K^a - b_\mu \Lambda_K^a + \omega_\mu{}^{ab}\Lambda_{Kb}\,,\\
\mathcal{D}_\mu \epsilon^i &= \partial_\mu \epsilon^i + \frac{1}{2} b_\mu \epsilon^i + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon^i - V_\mu^{ij}\epsilon_j\,,\\
\mathcal{D}_\mu \eta^i &= \partial_\mu \eta^i - \frac{1}{2} b_\mu \eta^i + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\eta^i - V_\mu^{ij}\eta_j\,.
\end{aligned} \tag{3.3}$$

Using the commutator expressions (B.1) we obtain the following expressions for the curvatures (terms proportional to vielbeins are underlined for later use):

$$\begin{aligned}
R_{\mu\nu}{}^a(P) &= 2\partial_{[\mu} e_{\nu]}{}^a + \underline{2\omega_{[\mu}{}^{ab} e_{\nu]b}} + \underline{2b_{[\mu} e_{\nu]}{}^a} - \frac{1}{2}\bar\psi_{[\mu}\gamma^a\psi_{\nu]}\,,\\
R_{\mu\nu}{}^{ab}(M) &= 2\partial_{[\mu}\omega_{\nu]}{}^{ab} + 2\omega_{[\mu}{}^{ac}\omega_{\nu]c}{}^b + \underline{8f_{[\mu}{}^{[a} e_{\nu]}{}^{b]}} + \mathrm{i}\,\bar\phi_{[\mu}\gamma^{ab}\psi_{\nu]}\,,\\
R_{\mu\nu}(D) &= 2\partial_{[\mu} b_{\nu]} - \underline{4f_{[\mu}{}^a e_{\nu]a}} - \mathrm{i}\,\bar\phi_{[\mu}\psi_{\nu]}\,,\\
R_{\mu\nu}{}^a(K) &= 2\partial_{[\mu} f_{\nu]}{}^a + 2\omega_{[\mu}{}^{ab} f_{\nu]b} - 2b_{[\mu} f_{\nu]}{}^a - \frac{1}{2}\bar\phi_{[\mu}\gamma^a\phi_{\nu]}\,,\\
R_{\mu\nu}{}^{ij}(V) &= 2\partial_{[\mu} V_{\nu]}{}^{ij} - 2V_{[\mu}{}^{k(i} V_{\nu]k}{}^{j)} - 3\,\mathrm{i}\,\bar\phi_{[\mu}^{(i}\psi_{\nu]}^{j)}\,,
\end{aligned} \tag{3.4}$$

$$
\begin{aligned}
R_{\mu\nu}{}^{i}(Q) &= 2\partial_{[\mu}\psi^{i}_{\nu]} + \frac{1}{2}\omega_{[\mu}{}^{ab}\gamma_{ab}\psi^{i}_{\nu]} + b_{[\mu}\psi^{i}_{\nu]} - 2V_{[\mu}{}^{ij}\psi_{\nu]j} + \underline{2\,\mathrm{i}\,\gamma_a\phi^{i}_{[\mu}e_{\nu]}{}^{a}}\,, \\
R_{\mu\nu}{}^{i}(S) &= 2\partial_{[\mu}\phi^{i}_{\nu]} + \frac{1}{2}\omega_{[\mu}{}^{ab}\gamma_{ab}\phi^{i}_{\nu]} - b_{[\mu}\phi^{i}_{\nu]} - 2V_{[\mu}{}^{ij}\phi_{\nu]j} - 2\,\mathrm{i}\,\gamma_a\psi^{i}_{[\mu}f_{\nu]}{}^{a}\,.
\end{aligned}
$$

Since the transformation laws given above satisfy the $F^2(4)$ superalgebra, we have a gauge theory of $F^2(4)$, but we do not have a gauge theory of diffeomorphisms of spacetime. This can only be realized if we take the spin connection as a composite field that depends on the vielbein. So far, we have it as an independent field.[4]

Furthermore, we see that the number of bosonic and fermionic degrees of freedom do not match. The gauge fields together have $96 + 64$ degrees of freedom. Therefore, we can not have a supersymmetric theory with invertible general coordinate transformations generated by the square of supersymmetry operations.

### 3.2 Constraints and their solutions

The solution to the problems described above is well known. In order to convert the $P$-gauge transformations into general coordinate transformations and to obtain irreducibility we need to impose curvature constraints and we have to introduce extra matter fields in the multiplet.

The constraints will define some gauge fields as dependent fields. The extra matter fields will also change the transformations of the gauge fields. In fact, we will have for the transformation (apart from the general coordinate transformations) of a general gauge field $h^I_\mu$:

$$
\delta_J(\epsilon^J)h^I_\mu = \partial_\mu\epsilon^I + \epsilon^J h_\mu{}^A f_{AJ}{}^I + \epsilon^J M_{\mu J}{}^I\,, \tag{3.5}
$$

where we use the index $I$ to denote all gauge transformations apart from general coordinate transformations, and an index $A$ includes the translations.

The last term depends on the matter fields, and its explicit form has to be determined below. But also the second term has contributions from matter fields. This is due to the fact that the structure 'functions' of the final algebra $f_{IJ}{}^K$ are modified from those of the $F^2(4)$ algebra which was used for (3.2). These extra terms lead also to modified curvatures

$$
\widehat{R}_{\mu\nu}{}^I = 2\partial_{[\mu}h_{\nu]}{}^I + h_\nu{}^B h_\mu{}^A f_{AB}{}^I - 2h_{[\mu}{}^J M_{\nu]J}{}^I\,. \tag{3.6}
$$

The commutator of two supersymmetry-transformations will also change. In particular we will find transformations with field-dependent parameters. They can

[4] One might think that the field equations can determine the spin connection as a dependent gauge field. This can indeed be done for the spin connection, but it is not known how to generalize this for the gauge fields of special (super)conformal symmetries, which we also want to be dependent gauge fields.



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be conveniently written as so-called covariant general coordinate transformations which are defined as

$$\delta_{cgct}(\xi) = \delta_{gct}(\xi) - \delta_I(\xi^\mu h_\mu{}^I)\,, \tag{3.7}$$

namely a combination of general coordinate transformations and all the other transformations whose parameter $\epsilon^I$ is replaced by $\xi^\mu h_\mu{}^I$. This takes a simpler form on fields of various types:

$$\begin{aligned}
\delta_{cgct}(\xi)e_\mu{}^a &= (\partial_\mu + b_\mu)\xi^a + \omega_\mu{}^{ab}\xi_b\,,\\
\delta_{cgct}(\xi)h^I_\mu &= -\xi^\nu \widehat{R}_{\mu\nu}{}^I - \xi^\nu h^J_\mu M_{\nu J}{}^I - \xi^\nu h^J_\mu e^a_\nu f_{aJ}{}^I\,,\\
\delta_{cgct}(\xi)\Phi &= \xi^\mu D_\mu \Phi\,.
\end{aligned} \tag{3.8}$$

The last terms for $B_{\mu\nu}$ are similar to the $M$-term for usual gauge fields in the second line. The last line holds for all covariant matter fields, including their covariant derivatives $D_a$, or covariant curvature tensors after changing the indices to local Lorentz indices.

We will consider the fünfbein as an invertible field. Then some of the curvatures in (3.4) are linear in some gauge fields. This is shown by the underlined terms in (3.4). Therefore, we can impose constraints on these curvatures that are solvable for these gauge fields. Such constraints are called conventional constraints, and imposing them reduces the Weyl multiplet, such that we get closer to an irreducible multiplet. The conventional constraints are[5]

$$\begin{aligned}
R_{\mu\nu}{}^a(P) &= 0 \ (50)\,,\\
e^\nu{}_b \widehat{R}_{\mu\nu}{}^{ab}(M) &= 0 \ (25)\,,\\
\gamma^\mu \widehat{R}_{\mu\nu}{}^i(Q) &= 0 \ (40)\,.
\end{aligned} \tag{3.9}$$

In brackets we denoted the number of restrictions each constraint imposes. These constraints are similar to those for other Weyl multiplets in 4 dimensions with $N = 1$ [19, 21], $N = 2$ [31] or $N = 4$ [28], or in 6 dimensions for the $(1,0)$ [22] or $(2,0)$ [29] Weyl multiplets.

In general one can add extra terms to the constraints (3.9), which just amount to redefinitions of the composite fields. By choosing suitable terms simplifications were obtained in 4 and 6 dimensions. In this case one could e.g. add a term $T_{\mu a}T^{ab}$ to the second constraint rendering all the constraints invariant under $S$-supersymmetry, but in 5 dimensions this turns out to be impossible. Therefore we keep the constraints as written above.

Due to these constraints the fields $\omega_\mu{}^{ab}$, $f_\mu{}^a$ and $\phi^i_\mu$ are no longer independent, but can be expressed in terms of the other fields. In order to write down the explicit

[5]Note that the third constraint implies that $\gamma_{[\mu\nu}\widehat{R}_{\rho\sigma]}{}^i(Q) = 0$.

solutions of these constraints, it is useful to extract the terms which have been underlined in (3.4). We define $\widehat{R}'$ as the curvatures without these terms. Formally,

$$\widehat{R}'_{\mu\nu}{}^{I} = \widehat{R}_{\mu\nu}{}^{I} + 2h^{J}_{[\mu}e^{a}_{\nu]}f_{aJ}{}^{I}\,, \tag{3.10}$$

where $f_{aJ}{}^{I}$ are the structure constants in the $F^2(4)$ algebra that define commutators of translations with other gauge transformations. Then the solutions to the constraints are

$$\begin{aligned}
\omega_{\mu}^{ab} &= 2e^{\nu[a}\partial_{[\mu}e_{\nu]}^{\ b]} - e^{\nu[a}e^{b]\sigma}e_{\mu c}\partial_{\nu}e_{\sigma}^{\ c} + 2e_{\mu}{}^{[a}b^{b]} - \frac{1}{2}\bar{\psi}^{[b}\gamma^{a]}\psi_{\mu} - \frac{1}{4}\bar{\psi}^{b}\gamma_{\mu}\psi^{a}\,,\\
\phi_{\mu}^{i} &= \frac{1}{3}\,\mathrm{i}\,\gamma^{a}\widehat{R}'_{\mu a}{}^{i}(Q) - \frac{1}{24}\,\mathrm{i}\,\gamma_{\mu}\gamma^{ab}\widehat{R}'_{ab}{}^{i}(Q)\,,\\
f_{\mu}^{a} &= \frac{1}{6}\mathcal{R}_{\mu}{}^{a} - \frac{1}{48}e_{\mu}{}^{a}\mathcal{R}\,, \qquad \mathcal{R}_{\mu\nu} \equiv \widehat{R}'_{\mu\rho}{}^{ab}(M)e_{a}{}^{\rho}e_{\nu b}\,, \qquad \mathcal{R} \equiv \mathcal{R}_{\mu}{}^{\mu}\,.
\end{aligned} \tag{3.11}$$

The constraints imply through Bianchi identities further relations between the curvatures. The Bianchi identities for $R(P)$ imply

$$\mathcal{R}_{\mu\nu} = \mathcal{R}_{\nu\mu}\,, \qquad e_{[\mu}{}^{a}\widehat{R}_{\nu\rho]}(D) = \widehat{R}_{[\mu\nu\rho]}{}^{a}(M)\,, \qquad \widehat{R}_{\mu\nu}(D) = 0\,. \tag{3.12}$$

### 3.3 Adding matter fields

After imposing the constraints we are left with 21 bosonic and 24 fermionic degrees of freedom. The independent fields are those in the left upper part of table 3. These have to be completed with matter fields to obtain the full Weyl multiplet. We have already seen that there are two possibilities for a $D = 5$ Weyl multiplet with each $32 + 32$ degrees of freedom.

These are obtained by adding either the left lower corner or right lower corner of table 3. To obtain all the extra transformations we imposed the superconformal algebra, but at the same time allowing modifications of the algebra by field-dependent quantities. The techniques are the same as already used in 4 and 6 dimensions in [31, 28], and were described in detail in [22].

For the fields in the upper left corner, we now have to specify the extra parts $M$ in (3.5). This will in fact only apply to $Q$-supersymmetry. The other transformations are as in (3.2). The extra terms we can read already from the linearized rules in (2.11) and (2.14). The full supersymmetry transformations of these fields are

$$\begin{aligned}
\delta_{Q}e_{\mu}{}^{a} &= \frac{1}{2}\bar{\epsilon}\gamma^{a}\psi_{\mu}\,,\\
\delta_{Q}\psi_{\mu}^{i} &= \mathcal{D}_{\mu}\epsilon^{i} + \mathrm{i}\,\gamma\cdot T\gamma_{\mu}\epsilon^{i}\,,\\
\delta_{Q}V_{\mu}{}^{ij} &= -\frac{3}{2}\,\mathrm{i}\,\bar{\epsilon}^{(i}\phi_{\mu}^{j)} + \mathrm{i}\,\bar{\epsilon}^{(i}\gamma\cdot T\psi_{\mu}^{j)} + 4\bar{\epsilon}^{(i}\gamma_{\mu}\chi^{j)}\,,\\
\delta_{Q}b_{\mu} &= \frac{1}{2}\,\mathrm{i}\,\bar{\epsilon}\phi_{\mu} - 2\bar{\epsilon}\gamma_{\mu}\chi\,,
\end{aligned} \tag{3.13}$$

where $\mathcal{D}_\mu\epsilon$ is given in (3.3). The fields $T_{ab}$ and $\chi^i$, and a further field $D$ that appears in their transformation laws (see below) are independent fields in the Standard Weyl multiplet, but not in the Dilaton Weyl multiplet. There, they are given by expressions that are the non-linear extensions of (2.13):

$$
\begin{aligned}
T_{ab} &= \frac{1}{8}\sigma^{-2}\left(\sigma\widehat{F}_{ab} - \frac{1}{6}\varepsilon_{abcde}\widehat{H}^{edc} + \frac{1}{4}\,\mathrm{i}\,\bar{\psi}\gamma_{ab}\psi\right), \\
\chi^i &= \frac{1}{8}\,\mathrm{i}\,\sigma^{-1}\not{D}\psi^i + \frac{1}{16}\,\mathrm{i}\,\sigma^{-2}\not{D}\sigma\psi^i - \frac{1}{32}\sigma^{-2}\gamma\cdot\widehat{F}\psi^i + \\
&\quad + \frac{1}{4}\sigma^{-1}\gamma\cdot\underline{T}\psi^i + \frac{1}{32}\,\mathrm{i}\,\sigma^{-3}\psi_j\bar{\psi}^i\psi^j\,, \\
D &= \frac{1}{4}\sigma^{-1}\Box^c\sigma + \frac{1}{8}\sigma^{-2}(D_a\sigma)(D^a\sigma) - \frac{1}{16}\sigma^{-2}\widehat{F}^2 - \\
&\quad - \frac{1}{8}\sigma^{-2}\bar{\psi}\not{D}\psi - \frac{1}{64}\sigma^{-4}\bar{\psi}^i\psi^j\bar{\psi}_i\psi_j - 4\,\mathrm{i}\,\sigma^{-1}\bar{\psi}\underline{\chi} + \\
&\quad + \left(-\frac{26}{3}\underline{T_{ab}} + 2\sigma^{-1}\widehat{F}_{ab} + \frac{1}{4}\,\mathrm{i}\,\sigma^{-2}\bar{\psi}\gamma_{ab}\psi\right)\underline{T^{ab}}\,,
\end{aligned}
\tag{3.14}
$$

where the conformal d'alembertian is defined by

$$
\begin{aligned}
\Box^c\sigma \equiv D^aD_a\sigma = \left(\partial^a - 2b^a + \omega_b{}^{ba}\right)D_a\sigma - \frac{1}{2}\,\mathrm{i}\,\bar{\psi}_aD^a\psi - 2\sigma\bar{\psi}_a\gamma^a\underline{\chi} + \\
+ \frac{1}{2}\bar{\psi}_a\gamma^a\gamma\cdot\underline{T}\psi + \frac{1}{2}\bar{\phi}_a\gamma^a\psi + 2f_a{}^a\sigma\,,
\end{aligned}
\tag{3.15}
$$

and where the underlining indicates that these terms are dependent fields. We have not substituted these terms in the expression for $D$ for reasons of brevity.

The modification $M$ in (3.5) is the last term of the transformations of $\psi_\mu^i$, $V_\mu^{ij}$ and $b_\mu$. The second term in the transformation of $V_\mu^{ij}$ on the other hand is due to the fact that the structure constants have become structure functions, and in particular there appears a new $T$-dependent SU(2) transformation in the anti-commutator of two supersymmetries. We will give the full new algebra in section 4. The transformation rules for the matter fields[6] of the Weyl multiplets are as follows. For the Standard Weyl multiplet we have ($Q$ and $S$ supersymmetry)

$$
\begin{aligned}
\delta T_{ab} &= \frac{1}{2}\,\mathrm{i}\,\bar{\epsilon}\gamma_{ab}\chi - \frac{3}{32}\,\mathrm{i}\,\bar{\epsilon}\widehat{R}_{ab}(Q)\,, \\
\delta\chi^i &= \frac{1}{4}\epsilon^iD - \frac{1}{64}\gamma\cdot\widehat{R}^{ij}(V)\epsilon_j + \frac{1}{8}\,\mathrm{i}\,\gamma^{ab}\not{D}T_{ab}\epsilon^i - \frac{1}{8}\,\mathrm{i}\,\gamma^aD^bT_{ab}\epsilon^i - \\
&\quad - \frac{1}{4}\gamma^{abcd}T_{ab}T_{cd}\epsilon^i + \frac{1}{6}T^2\epsilon^i + \frac{1}{4}\gamma\cdot T\eta^i\,, \\
\delta D &= \bar{\epsilon}\not{D}\chi - \frac{5}{3}\,\mathrm{i}\,\bar{\epsilon}\gamma\cdot T\chi - \mathrm{i}\,\bar{\eta}\chi\,.
\end{aligned}
\tag{3.16}
$$

[6] As we have already seen, two of the extra fields in the Dilaton Weyl multiplet are actually gauge fields, rather than matter fields. However, we use uniformly 'matter fields' for them in this context to indicate that they are not gauging a symmetry of the superconformal algebra.

There are no explicit gauge fields here, as should be the case for 'matter', i.e. non-gauge fields. These are all hidden in the covariant derivatives and covariant curvatures. The covariant derivatives are for any matter field given by the rule

$$D_a\Phi = e_a^\mu \left(\partial_\mu - \delta_I(h_\mu^I)\right)\Phi\,. \tag{3.17}$$

The last term represents thus a sum over all transformations except general coordinate transformations, with parameters replaced by the corresponding gauge fields. In practice, the Lorentz transformations and SU(2) transformations follow directly from the index structure and lead to additions similar to those in (3.3). For the Weyl transformations there is a term $-w\, b_\mu\Phi$, where $w$ is the Weyl weight of the field that can be found in table 3, and then there remain the terms for $Q$ and $S$ supersymmetry. There are no $K$ transformations for any matter field in this paper.

The covariant curvatures are given by the general rule (3.6), e.g.

$$\begin{aligned}\widehat{R}_{\mu\nu}{}^i(Q) &= R_{\mu\nu}{}^i(Q) + 2\,\mathrm{i}\,\gamma\cdot T\gamma_{[\mu}\psi_{\nu]}^i\,,\\ \widehat{R}_{\mu\nu}{}^{ij}(V) &= R_{\mu\nu}{}^{ij}(V) - 8\bar{\psi}_{[\mu}^{(i}\gamma_{\nu]}\chi^{j)} - \mathrm{i}\,\bar{\psi}_{[\mu}^{(i}\gamma\cdot T\psi_{\nu]}^{j)}\,,\end{aligned} \tag{3.18}$$

where $R_{\mu\nu}{}^i(Q)$ and $R_{\mu\nu}{}^{ij}(V)$ are those given in (3.4). Note that for $\widehat{R}(V)$ there are corrections from modified structure functions as well as from $M$-dependent terms. Having all the matter field dependence, we can obtain further consequences of the curvature constraints. E.g. the Bianchi identity on $\widehat{R}(Q)$ gives:

$$\begin{aligned}\gamma\cdot\widehat{R}(S) &= \frac{16}{3}T\cdot\widehat{R}(Q)\,,\\ \gamma^\mu\widehat{R}_{\mu\nu}(S) &= \frac{1}{2}\,\mathrm{i}\,D^\mu\widehat{R}_{\mu\nu}(Q) + \frac{8}{3}\gamma_\nu T\cdot\widehat{R}(Q) + 4\gamma^b T^\mu{}_b\widehat{R}_{\nu\mu}(Q)\,,\\ \widehat{R}_{\mu\nu}(S) &= \mathrm{i}\,\not{D}\widehat{R}_{\mu\nu}(Q) - \mathrm{i}\,\gamma_{[\mu}D^\rho\widehat{R}_{\nu]\rho}(Q) + \frac{16}{3}\gamma_{\mu\nu}T\cdot\widehat{R}(Q) -\\ &\quad -16\gamma^b T^\rho{}_b\gamma_{[\mu}\widehat{R}_{\nu]\rho}(Q) + 3\gamma\cdot T\widehat{R}_{\mu\nu}(Q) +\\ &\quad +16T^\rho{}_{[\mu}\widehat{R}_{\nu]\rho}(Q)\,.\end{aligned} \tag{3.19}$$

Given these transformation rules, we can calculate the transformations of the dependent fields. Their transformation rules are now determined by their definition due to the constraints. An equivalent way of expressing this is that their transformation rules are modified w.r.t. (3.2), due to the non-invariance of the constraints under these transformations. We have chosen the constraints to be invariant under all bosonic symmetries without modifications. Therefore, only the $Q$- and $S$-supersymmetries of the dependent fields are modified to get invariant constraints. The new transformation of the spin connection is

$$\begin{aligned}\delta\omega_\mu{}^{ab} &= \frac{1}{2}\,\mathrm{i}\,\bar{\epsilon}\gamma^{ab}\phi_\mu - \frac{1}{2}\,\mathrm{i}\,\bar{\eta}\gamma^{ab}\psi_\mu -\\ &\quad -\mathrm{i}\,\bar{\epsilon}\gamma^{[a}\gamma\cdot T\gamma^{b]}\psi_\mu -\\ &\quad -\frac{1}{2}\bar{\epsilon}\gamma^{[a}\widehat{R}_\mu{}^{b]}(Q) - \frac{1}{4}\bar{\epsilon}\gamma_\mu\widehat{R}^{ab}(Q) - 4e_\mu{}^{[a}\bar{\epsilon}\gamma^{b]}\chi\,.\end{aligned} \tag{3.20}$$

The first line is the transformation as implied from the $F^2(4)$ algebra, see (3.2). The second line is due to the modification of the anti-commutator of two supersymmetries by a $T$-dependent Lorentz rotation. Finally, the last line contains the terms that go into the $M$ of (3.5). We give here for $\phi^i_\mu$ just the latter type of terms

$$\begin{aligned}\delta\phi^i_\mu = &\ldots - \frac{1}{12}\,\mathrm{i}\left\{\gamma^{ab}\gamma_\mu - \frac{1}{2}\gamma_\mu\gamma^{ab}\right\}\widehat{R}_{ab}{}^i{}_j(V)\epsilon^j + \\ &+\frac{1}{3}\left[\not{D}\gamma\cdot T\gamma_\mu - D_\mu\gamma\cdot T + \gamma_\mu\gamma^c D^a T_{ac}\right]\epsilon^i + \\ &+\mathrm{i}\left[-\gamma_\mu(\gamma\cdot T)^2 + 4\gamma_c T_\mu{}^c\gamma\cdot T + 16\gamma_c T^{cd}T_{\mu d} - 4\gamma_\mu T^2\right]\epsilon^i + \\ &+\frac{1}{3}\,\mathrm{i}\left(8\gamma^b T_{\mu b} - \gamma_\mu\gamma\cdot T\right)\eta^i\,. \end{aligned} \tag{3.21}$$

We will not need the transformations for the field $f_\mu{}^a$, except the transformation of $f_a{}^a$ under $S$, since this term appears in the conformal d'alembertian. We only give its $M$-dependent $S$-transformation

$$\delta_S f_a{}^a = -5\,\mathrm{i}\,\bar{\eta}\chi\,. \tag{3.22}$$

We also used the transformation of the following curvatures:

$$\begin{aligned}\delta\widehat{R}_{ab}{}^i(Q) = &\frac{1}{6}\left(\gamma_{ab}{}^{cd} - \gamma^{cd}\gamma_{ab} - \frac{1}{2}\gamma_{ab}\gamma^{cd}\right)\widehat{R}_{cd}{}^i{}_j(V)\epsilon^j + \frac{1}{4}\widehat{R}_{ab}{}^{cd}(M)\gamma_{cd}\epsilon^i + \\ &+2\,\mathrm{i}\left(D_{[a}\gamma\cdot T\gamma_{b]} - \frac{1}{3}D_{[a}\gamma_{b]}\gamma\cdot T - \right. \\ &\qquad\left. - \frac{1}{3}\gamma_{[a}\not{D}\gamma\cdot T\gamma_{b]} - \frac{1}{3}\gamma_{ab}D_c\gamma_d T^{cd}\right)\epsilon^i\,, \end{aligned} \tag{3.23}$$

$$\begin{aligned}\delta\widehat{R}_{ab}{}^{ij}(V) = &-\frac{3}{2}\,\mathrm{i}\,\bar{\epsilon}^{(i}\widehat{R}_{ab}{}^{j)}(S) - 8\bar{\epsilon}^{(i}\gamma_{[a}D_{b]}\chi^{j)} + \mathrm{i}\,\bar{\epsilon}^{i}\gamma\cdot T\widehat{R}_{ab}{}^{j)}(Q) + \\ &+8\,\mathrm{i}\,\bar{\epsilon}^{(i}\gamma_{[a}\gamma\cdot T\gamma_{b]}\chi^{j)} + \frac{3}{2}\,\mathrm{i}\,\bar{\eta}^{(i}\widehat{R}_{ab}{}^{j)}(Q) + 8\,\mathrm{i}\,\bar{\eta}^{(i}\gamma_{ab}\chi^{j)}\,. \end{aligned}$$

The $Q$- and $S$-supersymmetry variations of the matter fields in the Dilaton Weyl multiplet are

$$\begin{aligned}\delta A_\mu &= -\frac{1}{2}\,\mathrm{i}\,\sigma\bar{\epsilon}\psi_\mu + \frac{1}{2}\bar{\epsilon}\gamma_\mu\psi\,, \\ \delta B_{\mu\nu} &= \frac{1}{2}\sigma^2\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} + \frac{1}{2}\,\mathrm{i}\,\sigma\bar{\epsilon}\gamma_{\mu\nu}\psi + A_{[\mu}\delta(\epsilon)A_{\nu]}\,, \\ \delta\psi^i &= -\frac{1}{4}\gamma\cdot\widehat{F}\epsilon^i - \frac{1}{2}\,\mathrm{i}\,\not{D}\sigma\epsilon^i + \sigma\gamma\cdot\underline{T}\epsilon^i - \frac{1}{4}\,\mathrm{i}\,\sigma^{-1}\epsilon_j\bar{\psi}^i\psi^j + \sigma\eta^i\,, \\ \delta\sigma &= \frac{1}{2}\,\mathrm{i}\,\bar{\epsilon}\psi\,. \end{aligned} \tag{3.24}$$

The gauge fields $A_\mu$ and $B_{\mu\nu}$ have the additional symmetries

$$\begin{aligned}\delta A_\mu &= \partial_\mu\Lambda\,, \\ \delta B_{\mu\nu} &= 2\partial_{[\mu}\Lambda_{\nu]} - \frac{1}{2}\Lambda F_{\mu\nu}\,. \end{aligned} \tag{3.25}$$

Note that the dependence of the transformation rules for $A_\mu$ and $B_{\mu\nu}$ on $\psi_\mu$ and $A_\mu$ signal new terms in the algebra of supersymmetries and $U(1)$ transformations[7]. This algebra will be written in section 4. On the other hand, the $F$-term in $\delta B_{\mu\nu}$ should be interpreted as an $M$ term according to (3.5), and modifies the field strength accordingly. This leads to the following field strengths of these gauge fields

$$\begin{aligned}\widehat{F}_{\mu\nu} &= 2\partial_{[\mu}A_{\nu]} + \frac{1}{2}\,\mathrm{i}\,\sigma\bar{\psi}_{[\mu}\psi_{\nu]} - \bar{\psi}_{[\mu}\gamma_{\nu]}\psi\,,\\ \widehat{H}_{\mu\nu\rho} &= 3\partial_{[\mu}B_{\nu\rho]} - \frac{3}{4}\sigma^2\bar{\psi}_{[\mu}\gamma_\nu\psi_{\rho]} - \frac{3}{2}\,\mathrm{i}\,\sigma\bar{\psi}_{[\mu}\gamma_{\nu\rho]}\psi + \frac{3}{2}A_{[\mu}F_{\nu\rho]}\,.\end{aligned} \tag{3.26}$$

For the convenience of the reader we give their transformation rules:

$$\begin{aligned}\delta\widehat{F}_{ab} &= -\frac{1}{2}\,\mathrm{i}\,\sigma\bar{\epsilon}\widehat{R}_{ab}(Q) - \bar{\epsilon}\gamma_{[a}D_{b]}\psi + \,\mathrm{i}\,\bar{\epsilon}\gamma_{[a}\gamma\cdot\underline{T}\gamma_{b]}\psi + \,\mathrm{i}\,\bar{\eta}\gamma_{ab}\psi\,,\\ \delta\widehat{H}_{abc} &= -\frac{3}{4}\sigma^2\bar{\epsilon}\gamma_{[a}\widehat{R}_{bc]}(Q) + \frac{3}{2}\,\mathrm{i}\,\bar{\epsilon}\gamma_{[ab}D_{c]}\psi + \frac{3}{2}\,\mathrm{i}\,D_{[a}\sigma\bar{\epsilon}\gamma_{bc]}\psi -\\ &\quad -\frac{3}{2}\sigma\bar{\epsilon}\gamma_{[a}\gamma\cdot\underline{T}\gamma_{bc]}\psi - \frac{3}{2}\bar{\epsilon}\gamma_{[a}\widehat{F}_{bc]}\psi - \frac{3}{2}\sigma\bar{\eta}\gamma_{abc}\psi\,.\end{aligned} \tag{3.27}$$

Finally, we give the Bianchi identities for these two curvatures

$$\begin{aligned}D_{[a}\widehat{F}_{bc]} &= \frac{1}{2}\bar{\psi}\gamma_{[a}\widehat{R}_{bc]}(Q)\,,\\ D_{[a}\widehat{H}_{bcd]} &= \frac{3}{4}\widehat{F}_{[ab}\widehat{F}_{cd]}\,.\end{aligned} \tag{3.28}$$

This finishes our discussion of the Standard and Dilaton Weyl multiplets. The final results for these multiplets have been collected in section 4. In the next section we will explain the connection between the two multiplets.

## 4. Results for the two Weyl multiplets

For the convenience of the reader we collect in this section the essential results of the previous sections, and give the supersymmetry algebra, which is modified by field-dependent terms. The transformation under dilatation is for each field $\delta_D\Phi = w\Lambda_D\Phi$, where the Weyl weight $w$ can be found in table (3). The Lorentz, and SU(2) transformations are evident from the index structure, and our normalizations can be found in (3.2).

### 4.1 The Standard Weyl multiplet

The $Q$- and $S$-supersymmetry and $K$-transformation rules for the independent fields of the Standard Weyl multiplet are

$$\delta e_\mu{}^a = \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu\,,$$

[7] The $A_{[\mu}\psi_{\nu]}$ term in $\delta B_{\mu\nu}$ is an extension of (3.5) that occurs for antisymmetric tensor gauge fields.

$$
\begin{aligned}
\delta\psi_\mu^i &= \mathcal{D}_\mu\epsilon^i + \mathrm{i}\,\gamma\cdot T\gamma_\mu\epsilon^i - \mathrm{i}\,\gamma_\mu\eta^i\,,\\
\delta V_\mu{}^{ij} &= -\frac{3}{2}\,\mathrm{i}\,\bar\epsilon^{(i}\phi_\mu^{j)} + 4\bar\epsilon^{(i}\gamma_\mu\chi^{j)} + \mathrm{i}\,\bar\epsilon^{(i}\gamma\cdot T\psi_\mu^{j)} + \frac{3}{2}\,\mathrm{i}\,\bar\eta^{(i}\psi_\mu^{j)}\,,\\
\delta T_{ab} &= \frac{1}{2}\,\mathrm{i}\,\bar\epsilon\gamma_{ab}\chi - \frac{3}{32}\,\mathrm{i}\,\bar\epsilon\widehat{R}_{ab}(Q)\,,\\
\delta\chi^i &= \frac{1}{4}\epsilon^i D - \frac{1}{64}\gamma\cdot\widehat{R}^{ij}(V)\epsilon_j + \frac{1}{8}\,\mathrm{i}\,\gamma^{ab}\not{D}T_{ab}\epsilon^i - \frac{1}{8}\,\mathrm{i}\,\gamma^a D^b T_{ab}\epsilon^i -\\
&\quad -\frac{1}{4}\gamma^{abcd}T_{ab}T_{cd}\epsilon^i + \frac{1}{6}T^2\epsilon^i + \frac{1}{4}\gamma\cdot T\eta^i\,,\\
\delta D &= \bar\epsilon\not{D}\chi - \frac{5}{3}\,\mathrm{i}\,\bar\epsilon\gamma\cdot T\chi - \mathrm{i}\,\bar\eta\chi\,,\\
\delta b_\mu &= \frac{1}{2}\,\mathrm{i}\,\bar\epsilon\phi_\mu - 2\bar\epsilon\gamma_\mu\chi + \frac{1}{2}\,\mathrm{i}\,\bar\eta\psi_\mu + 2\Lambda_{K\mu}\,.
\end{aligned}
\tag{4.1}
$$

The covariant derivative $\mathcal{D}_\mu\epsilon$ is given in (3.3). For other covariant derivatives, see the general rule (3.17), with more explanation below that equation. The covariant curvatures $\widehat{R}(Q)$ and $\widehat{R}(V)$ are given explicitly in (3.18). The expressions for the dependent fields are given in (3.11), where the prime indicates the omission of the underlined terms in (3.4).

## 4.2 The Dilaton Weyl multiplet

The Dilaton Weyl multiplet contains two extra gauge transformations: the gauge transformations of $A_\mu$ with parameter $\Lambda$ and those of $B_{\mu\nu}$ with parameter $\Lambda_\mu$. The transformation of the fields are given by:

$$
\begin{aligned}
\delta e_\mu{}^a &= \frac{1}{2}\bar\epsilon\gamma^a\psi_\mu\,,\\
\delta\psi_\mu^i &= \mathcal{D}_\mu\epsilon^i + \mathrm{i}\,\gamma\cdot\underline{T}\gamma_\mu\epsilon^i - \mathrm{i}\,\gamma_\mu\eta^i\,,\\
\delta V_\mu{}^{ij} &= -\frac{3}{2}\,\mathrm{i}\,\bar\epsilon^{(i}\phi_\mu^{j)} + 4\bar\epsilon^{(i}\gamma_\mu\underline{\chi^{j)}} + \mathrm{i}\,\bar\epsilon^{(i}\gamma\cdot\underline{T}\psi_\mu^{j)} + \frac{3}{2}\,\mathrm{i}\,\bar\eta^{(i}\psi_\mu^{j)}\,,\\
\delta A_\mu &= -\frac{1}{2}\,\mathrm{i}\,\sigma\bar\epsilon\psi_\mu + \frac{1}{2}\bar\epsilon\gamma_\mu\psi + \partial_\mu\Lambda\,,\\
\delta B_{\mu\nu} &= \frac{1}{2}\sigma^2\bar\epsilon\gamma_{[\mu}\psi_{\nu]} + \frac{1}{2}\,\mathrm{i}\,\sigma\bar\epsilon\gamma_{\mu\nu}\psi + A_{[\mu}\delta(\epsilon)A_{\nu]} + 2\partial_{[\mu}\Lambda_{\nu]} - \frac{1}{2}\Lambda F_{\mu\nu}\,,\\
\delta\psi^i &= -\frac{1}{4}\gamma\cdot\widehat{F}\epsilon^i - \frac{1}{2}\,\mathrm{i}\,\not{D}\sigma\epsilon^i + \sigma\gamma\cdot\underline{T}\epsilon^i - \frac{1}{4}\,\mathrm{i}\,\sigma^{-1}\epsilon_j\bar\psi^i\psi^j + \sigma\eta^i\,,\\
\delta\sigma &= \frac{1}{2}\,\mathrm{i}\,\bar\epsilon\psi\,,\\
\delta b_\mu &= \frac{1}{2}\,\mathrm{i}\,\bar\epsilon\phi_\mu - 2\bar\epsilon\gamma_\mu\underline{\chi} + \frac{1}{2}\,\mathrm{i}\,\bar\eta\psi_\mu + 2\Lambda_{K\mu}\,.
\end{aligned}
\tag{4.2}
$$

The covariant curvature of $A_\mu$ and $B_{\mu\nu}$ can be found in (3.26). The transformation of the dependent fields and the curvatures have been given in the previous section. We have underlined the fields $T_{ab}$ and $\chi^i$ to indicate that they are not independent fields but merely short-hand notations. The explicit expression for these fields in terms of fields of the Dilaton Weyl multiplet are given in (3.14).

### 4.3 Modified superconformal algebra

Finally, we present the 'soft' algebra that these Weyl multiplets realize. This is the algebra that all matter multiplets will have to satisfy, apart from possibly additional transformations under which the fields of the Weyl multiplets do not transform, and possibly field equations if these matter multiplets are on-shell.

The full commutator of two supersymmetry transformations is

$$\begin{aligned}[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = {} & \delta_{cgct}(\xi_3^\mu) + \delta_M(\lambda_3^{ab}) + \delta_S(\eta_3) + \delta_U(\lambda_3^{ij}) + \\ & + \delta_K(\Lambda^a_{K3}) + \delta_{U(1)}(\Lambda_3) + \delta_B(\Lambda_{3\mu}) \,. \end{aligned} \tag{4.3}$$

The covariant general coordinate transformations have been defined in (3.7). The last two terms appear obviously only in the Dilaton Weyl multiplet formulation. The parameters appearing in (4.3) are

$$\begin{aligned}
\xi_3^\mu &= \frac{1}{2}\bar\epsilon_2\gamma_\mu\epsilon_1 \,, \\
\lambda_3^{ab} &= -\,\mathrm{i}\,\bar\epsilon_2\gamma^{[a}\gamma\cdot T\gamma^{b]}\epsilon_1 \,, \\
\lambda_3^{ij} &= \mathrm{i}\,\bar\epsilon_2^{(i}\gamma\cdot T\epsilon_1^{j)} \,, \\
\eta_3^i &= -\frac{9}{4}\,\mathrm{i}\;\bar\epsilon_2\epsilon_1\chi^i + \frac{7}{4}\,\mathrm{i}\;\bar\epsilon_2\gamma_c\epsilon_1\gamma^c\chi^i + \\
&\quad +\frac{1}{4}\,\mathrm{i}\;\bar\epsilon_2^{(i}\gamma_{cd}\epsilon_1^{j)}\left(\gamma^{cd}\chi_j + \frac{1}{4}\,\widehat{R}^{cd}{}_j(Q)\right) \,, \\
\Lambda^a_{K3} &= -\frac{1}{2}\bar\epsilon_2\gamma^a\epsilon_1 D + \frac{1}{96}\bar\epsilon_2^i\gamma^{abc}\epsilon_1^j\widehat{R}_{bcij}(V) + \\
&\quad + \frac{1}{12}\,\mathrm{i}\,\bar\epsilon_2\left(-5\gamma^{abcd}D_bT_{cd} + 9D_bT^{ba}\right)\epsilon_1 + \\
&\quad + \bar\epsilon_2\left(\gamma^{abcde}T_{bc}T_{de} - 4\gamma^cT_{cd}T^{ad} + \frac{2}{3}\gamma^aT^2\right)\epsilon_1 \,, \\
\Lambda_3 &= -\frac{1}{2}\,\mathrm{i}\,\sigma\bar\epsilon_2\epsilon_1 \,, \\
\Lambda_{3\mu} &= -\frac{1}{2}\sigma^2\xi_{3\mu} - \frac{1}{2}A_\mu\Lambda_3 \,.
\end{aligned} \tag{4.4}$$

For the $Q, S$ commutators we find the following algebra:

$$\begin{aligned}
[\delta_S(\eta), \delta_Q(\epsilon)] &= \delta_D(\tfrac{1}{2}\,\mathrm{i}\,\bar\epsilon\eta) + \delta_M(\tfrac{1}{2}\,\mathrm{i}\,\bar\epsilon\gamma^{ab}\eta) + \delta_U(-\tfrac{3}{2}\,\mathrm{i}\,\bar\epsilon^{(i}\eta^{j)}) + \delta_K(\Lambda^a_{3K}) \,, \\
[\delta_S(\eta_1), \delta_S(\eta_2)] &= \delta_K(\tfrac{1}{2}\bar\eta_2\gamma^a\eta_1) \,.
\end{aligned} \tag{4.5}$$

with

$$\Lambda^a_{3K} = \frac{1}{6}\bar\epsilon\left(\gamma\cdot T\gamma_a - \frac{1}{2}\gamma_a\gamma\cdot T\right)\eta \,. \tag{4.6}$$

The commutator of $Q$ and $U(1)$ transformations is given by

$$[\delta(\epsilon), \delta(\Lambda)] = \delta_B\left(-\frac{1}{2}\Lambda\delta(\epsilon)A_\mu\right) . \tag{4.7}$$

This concludes our description of the Standard and Dilaton Weyl multiplets.

## 5. Connection between the Weyl multiplets

In the previous section we have shown that the Standard and Dilaton Weyl multiplets can be related to each other by expressing the fields of the Standard Weyl multiplet in terms of those of the Dilaton Weyl multiplet (see (3.14)). It is known that in 6 dimensions the coupling of an on-shell selfdual tensor multiplet to the $D = 6$ Standard Weyl multiplet leads to a $D = 6$ Dilaton Weyl multiplet [22]. Since in 5 dimensions a tensor multiplet is dual to a vector multiplet, it is natural to consider the coupling of a vector multiplet to the Standard Weyl multiplet. Since the Standard Weyl multiplet has no dilaton we must consider the improved vector multiplet. We will take the vector multiplet off-shell to simplify the higher-order fermion terms.

### 5.1 The improved vector multiplet

We will first consider the improved vector multiplet in a flat background, i.e. no coupling to conformal supergravity. Our starting point is the lagrangian corresponding to an off-shell vector multiplet:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\bar{\psi}\not{\partial}\psi - \frac{1}{2}(\partial\sigma)^2 + Y^{ij}Y_{ij}\,. \tag{5.1}$$

The action corresponding to this lagrangian is invariant under the off-shell supersymmetries

$$\begin{aligned}
\delta A_\mu &= \frac{1}{2}\bar{\epsilon}\gamma_\mu\psi\,,\\
\delta Y^{ij} &= -\frac{1}{2}\bar{\epsilon}^{(i}\not{\partial}\psi^{j)}\,,\\
\delta\psi^i &= -\frac{1}{4}\gamma\cdot F\epsilon^i - \frac{1}{2}\,\mathrm{i}\,\not{\partial}\sigma\epsilon^i - Y^{ij}\epsilon_j\,,\\
\delta\sigma &= \frac{1}{2}\,\mathrm{i}\,\bar{\epsilon}\psi\,.
\end{aligned} \tag{5.2}$$

The action has the wrong Weyl weight to be scale invariant. We therefore improve it by multiplying all terms with the dilaton. This requires additional cubic terms in the action to keep it invariant under supersymmetry. We thus obtain the lagrangian for the improved vector multiplet:

$$\begin{aligned}
\mathcal{L} = &-\frac{1}{4}\sigma F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\sigma\bar{\psi}\not{\partial}\psi - \frac{1}{2}\sigma(\partial\sigma)^2 + \sigma Y^{ij}Y_{ij} -\\
&-\frac{1}{8}\,\mathrm{i}\,\bar{\psi}\gamma\cdot F\psi - \frac{1}{24}\varepsilon_{\mu\nu\lambda\rho\sigma}A^\mu F^{\nu\lambda}F^{\rho\sigma} - \frac{1}{2}\,\mathrm{i}\,\bar{\psi}^i\psi^j Y_{ij}\,.
\end{aligned} \tag{5.3}$$

If we define the following

$$S^{ij} = 2\sigma Y^{ij} - \frac{1}{2}\,\mathrm{i}\,\bar{\psi}^i\psi^j\,,$$

$$
\begin{aligned}
\Gamma^i &= \mathrm{i}\,\sigma \not{\partial}\psi^i + \frac{1}{2}\,\mathrm{i}\,\not{\partial}\sigma\psi^i - \frac{1}{4}\gamma\cdot F\psi^i + Y^{ij}\psi_j\,,\\
C &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\bar{\psi}\not{\partial}\psi + \sigma\Box\sigma + \frac{1}{2}(\partial\sigma)^2 + Y^{ij}Y_{ij}\,,\\
H_a &= -\frac{1}{8}\varepsilon_{abcde}F^{bc}F^{de} - \partial^b\left(-\sigma F_{ba} - \frac{1}{4}\,\mathrm{i}\,\bar{\psi}\gamma_{ba}\psi\right)\,,\\
G_{abc} &= \partial_{[a}F_{bc]}\,,
\end{aligned}
\tag{5.4}
$$

then the equations of motion and the Bianchi identity corresponding to this lagrangian are given by

$$
0 = S^{ij} = \Gamma^i = C = H_a = G_{abc}\,. \tag{5.5}
$$

### 5.2 Coupling to the Standard Weyl multiplet

Next, we consider the coupling of the improved vector multiplet to the Standard Weyl multiplet. The transformation rules for the fields of the off-shell vector multiplet can be found by imposing the superconformal algebra (4.3). We thus find the following $Q$- and $S$-transformation rules:

$$
\begin{aligned}
\delta A_\mu &= -\frac{1}{2}\,\mathrm{i}\,\sigma\bar{\epsilon}\psi_\mu + \frac{1}{2}\bar{\epsilon}\gamma_\mu\psi\,,\\
\delta Y^{ij} &= -\frac{1}{2}\bar{\epsilon}^{(i}\not{D}\psi^{j)} + \frac{1}{2}\,\mathrm{i}\,\bar{\epsilon}^{(i}\gamma\cdot T\psi^{j)} - 4\,\mathrm{i}\,\sigma\bar{\epsilon}^{(i}\chi^{j)} + \frac{1}{2}\,\mathrm{i}\,\bar{\eta}^{(i}\psi^{j)}\,,\\
\delta\psi^i &= -\frac{1}{4}\gamma\cdot\widehat{F}\epsilon^i - \frac{1}{2}\,\mathrm{i}\,\not{D}\sigma\epsilon^i + \sigma\gamma\cdot T\epsilon^i - Y^{ij}\epsilon_j + \sigma\eta^i\,,\\
\delta\sigma &= \frac{1}{2}\,\mathrm{i}\,\bar{\epsilon}\psi\,,
\end{aligned}
\tag{5.6}
$$

where the covariant curvature is

$$
\widehat{F}_{\mu\nu} = 2\partial_{[\mu}A_{\nu]} + \frac{1}{2}\,\mathrm{i}\,\sigma\bar{\psi}_{[\mu}\psi_{\nu]} - \bar{\psi}_{[\mu}\gamma_{\nu]}\psi\,. \tag{5.7}
$$

The supercovariant extension of the Bianchi identity reads

$$
0 = G_{abc} = D_{[a}\widehat{F}_{bc]} - \frac{1}{2}\bar{\psi}\gamma_{[a}\widehat{R}_{bc]}(Q)\,. \tag{5.8}
$$

The first term in the transformation of $A_\mu$, reflected also in the curvature, signals a modification of the supersymmetry algebra, as can be seen by comparing with the general rule (3.5):

$$
[\delta(\epsilon_1),\delta(\epsilon_2)] = \ldots + \delta_{U(1)}\left(\Lambda_3 = -\frac{1}{2}\,\mathrm{i}\,\sigma\bar{\epsilon}_2\epsilon_1\right)\,, \tag{5.9}
$$

where the dots indicate all the terms present for the fields of the Standard Weyl multiplet and where the last term is the gauge transformation of $A_\mu$. This $U(1)$ is not part of the superconformal algebra and has no effect on the fields of the Standard Weyl multiplet. This is similar to the central charge induced in vector multiplets in 4 dimensions.

Our next goal is to find the equations of motion for the improved vector multiplet. These equations of motion should be an extension of the flat spacetime results given in (5.4). One way to proceed is to first find the curved background extension of the flat spacetime action defined by (5.3) and next derive the equations of motion from this action. However, for our present purposes, it is sufficient to find the equations of motion only.

We want to identify the spinor $\psi^i$ of the vector multiplet with the spinor $\psi^i$ of the Dilaton Weyl multiplet. This is why we have given these two spinors the same name in the first place (see the footnote in subsection 2.3). Comparing the SU(2) triplet term in the supersymmetry transformations of the two spinors, see (3.24) and (5.6), we deduce that the constraint $S^{ij}$ does not get any corrections and we must have

$$S^{ij} = 2\sigma Y^{ij} - \frac{1}{2}\,\mathrm{i}\,\bar{\psi}^i\psi^j\,. \tag{5.10}$$

There are now two ways to proceed. One way is to make the transition to an on-shell vector multiplet by using (5.10) to eliminate the auxiliary field $Y^{ij}$ from the transformation rules (5.6). The commutator of two supersymmetry transformations would then only close modulo the equations of motion.

A more elegant way is to note that the equations of motion must transform into each other. By varying (5.10) under (5.6) we find

$$\delta S^{ij} = \mathrm{i}\,\bar{\epsilon}^{(i}\Gamma^{j)}\,, \tag{5.11}$$

where the supercovariant extension of $\Gamma^i$ is now given by

$$\begin{aligned}\Gamma^i = {}& \mathrm{i}\,\sigma \not{D}\psi^i + \frac{1}{2}\,\mathrm{i}\,\not{D}\sigma\psi^i - \frac{1}{4}\gamma\cdot\widehat{F}\psi^i + Y^{ij}\psi_j + \\ & +2\sigma\gamma\cdot T\psi^i - 8\sigma^2\chi^i\,.\end{aligned} \tag{5.12}$$

Varying this expression under (5.6) and using (5.8) leads to the other equations of motion. We find:

$$\delta\Gamma^i = -\frac{1}{2}\,\mathrm{i}\,\not{D}S^{ij}\epsilon_j - \frac{1}{2}\,\mathrm{i}\,\gamma\cdot H\epsilon^i + \frac{1}{2}C\epsilon^i - \gamma\cdot TS^{ij}\epsilon_j\,, \tag{5.13}$$

where the supercovariant generalizations of (5.4) are given by

$$\begin{aligned}C = {}& -\frac{1}{4}\widehat{F}_{ab}\widehat{F}^{ab} - \frac{1}{2}\bar{\psi}\not{D}\psi + \sigma\Box^c\sigma + \frac{1}{2}D^a\sigma D_a\sigma + Y^{ij}Y_{ij} + \\ & + \mathrm{i}\,\bar{\psi}\gamma\cdot T\psi - 16\,\mathrm{i}\,\sigma\bar{\psi}\chi - \frac{104}{3}\sigma^2 T_{ab}T^{ab} + 8\sigma\widehat{F}_{ab}T^{ab} - 4\sigma^2 D\,,\\ H_a = {}& -\frac{1}{8}\varepsilon_{abcde}\widehat{F}^{bc}\widehat{F}^{de} - D^b\left(8\sigma^2 T_{ba} - \sigma\widehat{F}_{ba} - \frac{1}{4}\,\mathrm{i}\,\bar{\psi}\gamma_{ba}\psi\right)\,.\end{aligned} \tag{5.14}$$

The supercovariant equations of motion and Bianchi identity are then given by

$$0 = S^{ij} = \Gamma^i = C = H_a = G_{abc}\,. \tag{5.15}$$

### 5.3 Solving the equations of motion

In 6 dimensions, the equations of motion for an on-shell tensor multiplet coupled to the Standard Weyl multiplet can be used to eliminate the matter fields of the latter in terms of the matter fields of the Dilaton Weyl multiplet. Precisely the same happens here. First of all the equations of motion for $Y^{ij}$ can be used to eliminate this auxiliary field. Next, the equations of motion for $\psi^i$ and $\sigma$ can be used to solve for the fields $\chi^i$ and $D$, respectively. The expressions for these fields exactly coincide with the ones we found in (3.14).

The solution for the matter field $T_{ab}$ in terms of the fields of the Dilaton Weyl multiplet is more subtle. It requires that we first reinterpret the equation of motion for the vector field as the Bianchi identity for a two-form antisymmetric tensor gauge field $B_{\mu\nu}$. To be precise, we rewrite $H_a = 0$ from (5.14) as a Bianchi identity

$$D_{[a}\widehat{H}_{bcd]} = \frac{3}{4}\widehat{F}_{[ab}\widehat{F}_{cd]}\,, \tag{5.16}$$

where the three-form curvature $\widehat{H}_{abc}$ is defined by

$$-\frac{1}{6}\varepsilon_{abcde}\widehat{H}^{edc} = 8\sigma^2 T_{ab} - \sigma\widehat{F}_{ab} - \frac{1}{4}\,\mathrm{i}\,\bar{\psi}\gamma_{ab}\psi\,. \tag{5.17}$$

Note that the latter equation is just a rewriting of the relation (3.14) we found in section 3.

The Bianchi identity (5.16) can be solved in terms of an antisymmetric two-form gauge field $B_{\mu\nu}$. The superconformal algebra (5.9) imposes that such a field transforms under supersymmetry as follows:

$$\delta_Q B_{\mu\nu} = \frac{1}{2}\sigma^2\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} + \frac{1}{2}\,\mathrm{i}\,\sigma\bar{\epsilon}\gamma_{\mu\nu}\psi + A_{[\mu}\delta(\epsilon)A_{\nu]}\,. \tag{5.18}$$

In addition one finds that the field $B_{\mu\nu}$ transforms under a U(1) and a vector gauge transformation as follows

$$\delta B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]} - \frac{1}{2}\Lambda F_{\mu\nu}\,. \tag{5.19}$$

Furthermore, the commutator of two $Q$-transformations picks up a vector gauge transformation $\delta_B$ for the field $B_{\mu\nu}$:

$$\begin{aligned}
&[\delta(\epsilon_1), \delta(\epsilon_2)] = \ldots + \delta_{U(1)}(\Lambda_3) + \delta_B(\Lambda_{3\mu})\,,\\
&\Lambda_3 = -\frac{1}{2}\,\mathrm{i}\,\sigma\bar{\epsilon}_2\epsilon_1\,, \qquad \Lambda_{3\mu} = -\frac{1}{4}\sigma^2\bar{\epsilon}_2\gamma_\mu\epsilon_1 - \frac{1}{2}A_\mu\Lambda_3\,.
\end{aligned} \tag{5.20}$$

From the transformation rules (5.19) for $B_{\mu\nu}$ it follows that the supercovariant field strength $\widehat{H}_{\mu\nu\rho}$ is given by

$$\widehat{H}_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]} - \frac{3}{4}\sigma^2\bar{\psi}_{[\mu}\gamma_\nu\psi_{\rho]} - \frac{3}{2}\,\mathrm{i}\,\sigma\bar{\psi}_{[\mu}\gamma_{\nu\rho]}\psi + \frac{3}{2}A_{[\mu}F_{\nu\rho]}\,. \tag{5.21}$$

This field strength indeed satisfies the Bianchi identity (5.16).

| Dimension $D$ | # d.o.f. | Standard Weyl | Dilaton Weyl |
|---|---|---|---|
| 6 | 10 | $T^{+}_{abc}$ | $B_{\mu\nu}$ |
| 5 | 10 | $T_{ab}$ | $A_\mu\,,B_{\mu\nu}$ |
| 4 | 6 | $T_{ab}$ | $A_\mu\,,B_\mu$ |

**Table 6:** The two different formulations of the Weyl multiplet in $D = 4, 5, 6$.

We conclude that the connection between the Standard and Dilaton Weyl multiplets can be obtained by first coupling an improved vector multiplet to the Standard Weyl multiplet and, next, solving the equations of motion. To solve for the equation of motion for the vector field in terms of the matter field $T_{ab}$ one must first reinterpret this equation of motion as the Bianchi identity for an antisymmetric two-form gauge field.

## 6. Conclusions

In this work we have taken the first step in the superconformal tensor calculus by constructing the Weyl multiplets for $N = 2$ conformal supergravity theory in 5 dimensions.

First, we have applied the standard current multiplet procedure to the case of the $D = 5$ vector multiplet. An unconventional feature is that the corresponding energy-momentum tensor is neither traceless nor improvable to a traceless current. However, since one version of the Weyl multiplet contains a dilaton we could construct this linearized $32 + 32$ Dilaton Weyl multiplet from the current multiplet. We also pointed out that there exists a second ('Standard') Weyl multiplet without a dilaton, by comparing with similar Weyl multiplets in $D = 4$ and $D = 6$.

Next, we explained how the non-linear multiplets could be obtained by gauging the superconformal algebra $F^2(4)$. Finally, we discussed the relation between the two Weyl multiplets. We showed that the coupling of the Standard Weyl multiplet to an improved vector multiplet leads to the non-linear relation between the Standard and Dilaton Weyl multiplets.

The fact that there exist two different versions of conformal supergravity has been encountered before in 6 dimensions [22]. Table 6 suggests that the same feature might also occur in 4 dimensions. It seems plausible that in this case the coupling of a vector multiplet to the Standard Weyl multiplet will give a Dilaton Weyl multiplet containing two vectors. It would be interesting to see whether the Dilaton Weyl multiplet in 4 dimensions indeed exists, and how it can be used in matter couplings.

It is known, from the AdS/CFT correspondence, that there is a relation between $(1, 1)$, $D = 6$ gauged supergravity with 16 supercharges and the $N = 2, D = 5$ conformal supergravity with 8 supercharges. In fact, the precise relation between the Standard Weyl multiplet and the $(1, 1), D = 6$, or $F(4)$-gauged, supergravity [32] has



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been given in [5]. The $F(4)$-gauged supergravity contains a *massive* antisymmetric two-form tensor which, according to [5], corresponds to the $T_{ab}$ matter field of the Standard Weyl multiplet. It would be interesting to see whether the work of [5] can be extended such that the $F(4)$-gauged supergravity theory also gives rise to the Dilaton Weyl multiplet. One possibility is that, in order to achieve this, one should first replace the massive two-form of gauged supergravity by a *massless* one-form and two-form gauge field.

Finally, the results of this work will be our starting point for the construction of general supergravity/matter couplings in 5 dimensions. We hope to report about this in the nearby future.

**Note added:** A few days after we sent this paper to the bulletin board an interesting paper appeared on conformal supergravity in five dimensions [41] that has some overlap with our work. The authors of [41] also discuss the two versions of the Weyl multiplet. In addition they discuss the superconformal tensor calculus in five dimensions.



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## A. Notations and Conventions

The metric is $(-++++)$, and we use the following indices (spinor indices are always omitted)

$$\begin{aligned}
&\mu \; 0,\ldots,4 \text{ local spacetime}\,,\\
&a \; 0,\ldots,4 \text{ tangent spacetime}\,,\\
&i \quad 1,2 \quad \mathrm{SU}(2)\,.
\end{aligned} \tag{A.1}$$

The generators $U_{ij}$ of the $R$-symmetry group SU(2) are defined to be anti-hermitian and symmetric, i.e.

$$(U_i{}^j)^* = -U_j{}^i\,, \qquad U_{ij} = U_{ji}\,. \tag{A.2}$$

A symmetric traceless $U_i{}^j$ corresponds to a symmetric $U^{ij}$ since we lower or raise SU(2) indices using the $\varepsilon$-symbol, in NW–SE convention:

$$X^i = \varepsilon^{ij} X_j\,, \qquad X_i = X^j \varepsilon_{ji}\,, \qquad \varepsilon_{12} = -\varepsilon_{21} = \varepsilon^{12} = 1\,. \tag{A.3}$$

The actual value of $\varepsilon$ is here given as an example. It is in fact arbitrary as long as it is antisymmetric, $\varepsilon^{ij} = (\varepsilon_{ij})^*$ and $\varepsilon_{jk}\varepsilon^{ik} = \delta_j{}^i$.

The charge conjugation matrix $\mathcal{C}$ and $\mathcal{C}\gamma_a$ are antisymmetric. The matrix $\mathcal{C}$ is unitary and $\gamma_a$ is hermitian apart from the timelike one, which is anti-hermitian. The bar is the Majorana bar:

$$\bar\lambda^i = (\lambda^i)^T \mathcal{C}\,. \tag{A.4}$$

We define the charge conjugation operation on spinors as

$$(\lambda^i)^C \equiv \alpha^{-1} B^{-1} \varepsilon^{ij} (\lambda^j)^*\,, \qquad \bar\lambda^{iC} \equiv \overline{(\lambda^i)^C} = \alpha^{-1} \left(\bar\lambda^k\right)^* B \varepsilon^{ki}\,, \tag{A.5}$$

where $B = \mathcal{C}\gamma_0$, and $\alpha = \pm 1$ when one uses the convention that complex conjugation does not interchange the order of spinors, or $\alpha = \pm\,\mathrm{i}$ when it does. Symplectic Majorana spinors satisfy $\lambda = \lambda^C$. Charge conjugation acts on gamma matrices as $(\gamma_a)^C = -\gamma_a$, does not change the order of matrices, and works on matrices in SU(2) space as $M^C = \sigma_2 M^* \sigma_2$. Complex conjugation can then be replaced by charge conjugation, if for every bispinor one inserts a factor $-1$. Then, e.g. the expressions

$$\bar\lambda^i \gamma_\mu \lambda^j\,, \qquad \mathrm{i}\,\bar\lambda^i \lambda_i \tag{A.6}$$

are real for symplectic Majorana spinors. For more details, see e.g. [24].

When the SU(2) indices on spinors are omitted, northwest-southeast contraction is understood, e.g.

$$\bar\lambda \gamma^{(n)} \psi = \bar\lambda^i \gamma^{(n)} \psi_i\,, \tag{A.7}$$

where we have used the following notation

$$\gamma^{(n)} = \gamma^{a_1 \cdots a_n} = \gamma^{[a_1} \gamma^{a_2} \cdots \gamma^{a_n]}\,. \tag{A.8}$$

The anti-symmetrizations are always with unit strength. Changing the order of spinors in a bilinear leads to the following signs

$$\bar\psi^{(1)} \gamma_{(n)} \chi^{(2)} = t_n\ \bar\chi^{(2)} \gamma_{(n)} \psi^{(1)}\ , \qquad \begin{cases} t_n = -1 \text{ for } n = 2,3 \\ t_n = +1 \text{ for } n = 0,1 \end{cases} \tag{A.9}$$

where the labels (1) and (2) denote any SU(2) representation.

We frequently use the following Fierz rearrangement formulae

$$\psi_j \bar\lambda^i = -\frac{1}{4} \bar\lambda^i \psi_j - \frac{1}{4} \bar\lambda^i \gamma^a \psi_j \gamma_a + \frac{1}{8} \bar\lambda^i \gamma^{ab} \psi_j \gamma_{ab}\,, \qquad \bar\psi^{[i} \lambda^{j]} = -\frac{1}{2} \bar\psi \lambda \varepsilon^{ij}\,. \tag{A.10}$$

When one multiplies three spinor doublets, one should be able to write the result in terms of $(8 \times 7 \times 6)/3! = 56$ independent structures. From analyzing the representations, one can obtain that these are in the $(4,2)+(4,4)+(16,2)$ representations of $\overline{\mathrm{SO}(5)} \times \mathrm{SU}(2)$. They are

| $d=5$ | $m=1$ | $m=2$ |
|---|---|---|
| $n=0$ | 5 | $-20$ |
| $n=1$ | $-3$ | $-4$ |
| $n=2$ | 1 | 4 |
| $n=3$ | 1 | 4 |

**Table 7:** The coefficients $c_{n,m}$ in (A.17).

$$\begin{aligned} &\xi_j \bar{\xi}^j \xi^i = \gamma^a \xi_j \bar{\xi}^j \gamma_a \xi^i = \frac{1}{8} \gamma^{ab} \xi^i \bar{\xi} \gamma_{ab} \xi \,, \\ &\xi^{(k} \bar{\xi}^i \xi^{j)} \,, \\ &\xi_j \bar{\xi}^j \gamma_a \xi^i \,. \end{aligned} \tag{A.11}$$

The Levi–Civita tensor is real and satisfies

$$\varepsilon_{a_1 \ldots a_n b_1 \ldots b_p} \varepsilon^{a_1 \ldots a_n c_1 \ldots c_p} = -n! p! \delta_{[b_1}^{[c_1} \ldots \delta_{b_p]}^{c_p]} \,, \qquad \varepsilon^{\mu\nu\rho\sigma\tau} = e e_a^\mu e_b^\nu \cdots e_e^\tau \varepsilon^{abcde} \,. \tag{A.12}$$

We introduce the dual of a tensor as

$$\tilde{A}^{a_1 \ldots a_{5-n}} = \frac{1}{n!} \, \mathrm{i} \, \varepsilon_{a_1 \ldots a_{5-n} b_1 \ldots b_n} A^{b_n \ldots b_1} \,, \tag{A.13}$$

with the properties

$$\tilde{\tilde{A}} = A \,, \qquad \frac{1}{n!} A^{a_1 \ldots a_n} B_{a_1 \ldots a_n} = \frac{1}{n!} A \cdot B = \frac{1}{(n-5)!} \tilde{A} \cdot \tilde{B} \,, \tag{A.14}$$

where we have introduced the notation $A \cdot B$ that we use throughout the paper.

The product of all gamma matrices is proportional to the unit matrix in odd dimensions. We use

$$\gamma^{abcde} = \mathrm{i} \, \varepsilon^{abcde} \,. \tag{A.15}$$

This implies that the dual of a $(5-n)$-antisymmetric gamma matrix is the $n$-antisymmetric gamma matrix given by

$$\gamma_{a_1 \ldots a_n} = \frac{1}{(5-n)!} \, \mathrm{i} \, \varepsilon_{a_1 \ldots a_n b_1 \ldots b_{5-n}} \gamma^{b_{5-n} \ldots b_1} \,. \tag{A.16}$$

For convenience we will give a rule for calculating gamma-contractions like

$$\gamma^{(m)} \gamma_{(n)} \gamma_{(m)} = c_{n,m} \gamma_{(n)} \,, \tag{A.17}$$

where the constants $c_{n,m}$ are given for the most frequently used cases in table 7.

## B. The $D = 5$ superconformal algebra $F^2(4)$

There exist many varieties of superconformal algebras, when one allows for central charges [33, 34]. However, so far a suitable superconformal Weyl multiplet has only been constructed from those superconformal algebras[8] that appear in the Nahm's classification [36]. In that classification appears one exceptional algebra, which is $F(4)$. The particular real form that we need here is denoted by $F^2(4)$, see tables 5 and 6 in [24].

The commutation relations defining the $F^2(4)$ algebra are given by

$$\begin{aligned}
[P_a, M_{bc}] &= \eta_{a[b}P_{c]}\,, & [K_a, M_{bc}] &= \eta_{a[b}K_{c]}\,,\\
[D, P_a] &= P_a\,, & [D, K_a] &= -K_a\,,\\
\left[M_{ab}, M^{cd}\right] &= -2\delta_{[a}{}^{[c}M_{b]}{}^{d]}\,, & [P_a, K_b] &= 2(\eta_{ab}D + 2M_{ab})\,,\\
[M_{ab}, Q_{i\alpha}] &= -\frac{1}{4}(\gamma_{ab}Q_i)_\alpha\,, & [M_{ab}, S_{i\alpha}] &= -\frac{1}{4}(\gamma_{ab}S_i)_\alpha\,,\\
[D, Q_{i\alpha}] &= \frac{1}{2}Q_{i\alpha}\,, & [D, S_{i\alpha}] &= -\frac{1}{2}S_{i\alpha}\,,\\
[K_a, Q_{i\alpha}] &= \mathrm{i}\,(\gamma_a S_i)_\alpha\,, & [P_a, S_{i\alpha}] &= -\,\mathrm{i}\,(\gamma_a Q_i)_\alpha\,,\\
\{Q_{i\alpha}, Q_{j\beta}\} &= -\frac{1}{2}\varepsilon_{ij}(\gamma^a)_{\alpha\beta}P_a\,, & \{S_{i\alpha}, S_{j\beta}\} &= -\frac{1}{2}\varepsilon_{ij}(\gamma^a)_{\alpha\beta}K_a\,,\\
\{Q_{i\alpha}, S_{j\beta}\} &= -\frac{1}{2}\,\mathrm{i}\,\left(\varepsilon_{ij}C_{\alpha\beta}D + \varepsilon_{ij}(\gamma^{ab})_{\alpha\beta}M_{ab} + 3C_{\alpha\beta}U_{ij}\right)\,, &&\\
[Q_{i\alpha}, U_{kl}] &= \varepsilon_{i(k}Q_{l)\alpha}\,, & [S_{i\alpha}, U_{kl}] &= \varepsilon_{i(k}S_{l)\alpha}\,,\\
\left[U_{ij}, U^{kl}\right] &= 2\delta_{(i}{}^{(k}U_{j)}{}^{l)}\,. &&
\end{aligned} \tag{B.1}$$

The first six commutation relations define the bosonic conformal algebra $\mathrm{SO}(5,2)$.

## C. The current multiplet of Howe-Lindström

The supercurrent in 5 dimensions has been discussed before in the literature [25, 37]. The authors of [25] found a $40+40$ current multiplet that couples to a $(32+32)$ plus $(8+8)$ reducible field multiplet. It turns out that also the current multiplet itself is reducible. More precisely, the $40+40$ multiplet of [25] reduces to our $32+32$ multiplet and an additional $8+8$ multiplet.

The $40+40$ multiplet has all the currents of the $32+32$ multiplet in table 2, except the current $b_{\mu\nu}$. In addition it contains the currents which we present in table 8.

The multiplet of [25] is generated by varying $\sigma^2$ under supersymmetry until closure is reached and in this way it produces $40+40$ components. In particular, we find that the transformations of the fields in table 8 are given by:

$$\begin{aligned}
\delta_Q c &= \mathrm{i}\,\bar\epsilon\chi\,,\\
\delta_Q \chi^i &= -\frac{1}{4}\,\mathrm{i}\,\partial\!\!\!/ c\epsilon^i + \frac{1}{4}y^{ij}\epsilon_j + \frac{1}{4}\,\mathrm{i}\,\gamma^\mu v_\mu^{ij}\epsilon_j - \frac{1}{4}\gamma\cdot n\epsilon^i\,,
\end{aligned}$$

[8] One notable case is the 10 dimensional Weyl multiplet [35], that is not based on a known algebra.

| Current | SU(2) | $w$ | # d.o.f. | Expression | Redefined | $w$ |
|---|---|---|---|---|---|---|
| $c$ | 1 | 2 | 1 | $\sigma^2$ | $\widehat{c}$ | 4 |
| $y^{(ij)}$ | 3 | 3 | 3 | $\frac{1}{2}\,\mathrm{i}\,\bar{\psi}^i\psi^j$ | $y^{ij}$ | 3 |
| $n_{[\mu\nu]}$ | 1 | 1 | 10 | $\sigma F_{\mu\nu} + \frac{1}{8}\,\mathrm{i}\,\bar{\psi}\gamma_{\mu\nu}\psi$ | $b_{[\mu\nu]} + \widehat{a}_\mu$ | 3 |
| $\chi^i$ | 2 | 5/2 | 8 | $\sigma\psi^i$ | $\widehat{\chi}^i$ | 7/2 |

**Table 8:** The extra fields of the $40 + 40$ current multiplet. The column "Redefined" indicates the currents after the field redefinitions (C.4). The field $n_{\mu\nu}$ is not a Noether current, but after the field redefinition (C.4) it gives the Noether currents $b_{[\mu\nu]}$ and $\widehat{a}_\mu$.

$$\begin{aligned}
\delta_Q y^{ij} &= \mathrm{i}\,\bar{\epsilon}^{(i}\,\mathrm{i}\,\gamma\cdot J^{j)} + 2\bar{\epsilon}^{(i}\not{\partial}\chi^{j)}\,,\\
\delta_Q n_{\mu\nu} &= \frac{1}{4}\bar{\epsilon}\gamma_{\mu\nu\lambda}J^\lambda + \bar{\epsilon}\partial_{[\mu}\gamma_{\nu]}\chi\,.
\end{aligned} \tag{C.1}$$

In addition the transformation of the supercurrent is also slightly changed w.r.t. (2.9) since it does not contain $b_{\mu\nu}$ but $n_{\mu\nu}$:

$$\delta_Q J^i_\mu = -\frac{1}{2}\,\mathrm{i}\,\gamma^\nu\theta_{\nu\mu}\epsilon^i - \mathrm{i}\,\gamma_{[\lambda}\partial^\lambda v^{ij}_{\mu]}\epsilon_j - \frac{1}{2}a_\mu\epsilon^i + \frac{1}{4}\,\mathrm{i}\,\varepsilon_{\mu\nu\lambda\rho\sigma}\gamma^\nu\partial^\lambda n^{\rho\sigma}\epsilon^i\,. \tag{C.2}$$

Comparing this with (2.9) we see that the relation between $b_{\mu\nu}$ and $n_{\mu\nu}$ is given by

$$b_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho\sigma}\partial^\lambda n^{\rho\sigma}\,. \tag{C.3}$$

The $32 + 32$ components now transform only among themselves according to (2.9). To demonstrate full reducibility of the $40 + 40$ multiplet we make the following field redefinitions:

$$\begin{aligned}
\widehat{c} &= \theta_\mu{}^\mu - \Box c\,,\\
\widehat{\chi}^i &= \mathrm{i}\,\gamma\cdot J^i - 2\,\mathrm{i}\not{\partial}\chi^i\,,\\
\widehat{a}_\mu &= a_\mu - 2\partial^\nu n_{\nu\mu}\,.
\end{aligned} \tag{C.4}$$

Together with the field $y^{ij}$ these fields form an $8 + 8$ multiplet, see table 8, transforming only among themselves according to

$$\begin{aligned}
\delta_Q\widehat{c} &= \frac{1}{2}\bar{\epsilon}\not{\partial}\widehat{\chi}\,,\\
\delta_Q\widehat{\chi}^i &= \frac{1}{2}\widehat{c}\epsilon^i - \frac{1}{2}\,\mathrm{i}\not{\partial}y^{ij}\epsilon_j - \frac{1}{2}\,\mathrm{i}\,\not{\widehat{a}}\epsilon^i\,,\\
\delta_Q y^{ij} &= \mathrm{i}\,\bar{\epsilon}^{(i}\widehat{\chi}^{j)}\,,\\
\delta_Q\widehat{a}_\mu &= -\frac{1}{2}\,\mathrm{i}\,\bar{\epsilon}\gamma_{\mu\nu}\partial^\nu\widehat{\chi}\,.
\end{aligned} \tag{C.5}$$

As one would suspect from the field content, the $8+8$ current multiplet (C.5) is conjugate to the off-shell vector multiplet (5.2), in the same way as the 32+32 current multiplet (2.9) is conjugate to the Dilaton Weyl multiplet (2.11). We expect that this multiplet can be used as one of the compensating multiplets in the construction of the $48 + 48$ off-shell [38, 37] ($= 8 + 8$ on-shell [39]) $D = 5$ Poincaré multiplet.



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